Tunable Pulse-Shaping Wave Digital Filter Networks

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This paper describes the design of Wave Digital Filters (WDFs) that perform basic functions of classical analogue pulse-shaping networks in radiation spectroscopy. The resulting WDF structures realise CR-RC type and semi-Gaussian shaping of digitised, preamplifier signals. The basic section of the WDF is derived from a resistively terminated analogue network and the same configuration is used for the whole structure giving a modular design. Each section of the WDF has a single tunable multiplier coefficient that controls the parameters of the output pulse. In comparison to other digital filters, WDFs have the advantages of highly modular structures, minimum tunable coefficients and direct correspondence with analogue pulse-shaping network parameters.

Keywords: Wave digital filter, pulse-shaping network, radiation spectroscopy.

1. Introduction

Analogue filters are traditionally used for pulse-processing in radiation spectroscopy systems. Also known as amplifiers, these circuits are useful due to the frequency selective properties that mould and shape the pulses of the preamplifier. The required shaping may involve a simple shortening of the long tail that characterises many preamplifier signals and reducing its noise content to allow for a more precise determination of the amplitude. On the other hand, the shaping may require more complex operations to obtain the cusp, Gaussian or triangular shapes. Combinations of passive low order filters are found to be practical in providing basic CR-RC type and semi-Gaussian shaping while active filters are used for semi-triangular shaping. The cusp and true Gaussian shaping is not physically realisable, while true triangular shaping is realisable but not in a cost-effective way [1]–[2].

More recently, digital systems were introduced as an alternative to classical analogue spectroscopy systems [3]–[4]. These systems offer the advantages as other digital systems over their analogue counterparts in terms of design flexibility and reliability. Within the digital systems, digital filters are used to shape the sampled and digitised preamplifier pulses. These filters are either Finite Impulse Response (FIR) or Infinite Impulse Response (HR) having the required transfer characteristics [5]–[7]. However, these realisations use structures with non-optimum number of coefficients. In order to change the transfer function of a filter section, modifications to all the coefficients are required.

Wave Digital Filters (WDFs) are IIR filters as the transfer function is derived from passive analogue filters using a bilinear transformation. There are many types of WDFs based on the analogue filters that are used as the reference, e.g., ladder, lattice and unit element WDFs. In addition to the transfer function, a WDF mimics the structure of the analogue reference filter from which it is derived. The WDF retains the good sensitivity properties of a well-designed analogue filter which results in structures with reduced coefficient wordlength requirements, good dynamic range and excellent stability. WDFs also have the power complementary transfer function property. The same structure can be used to realise the lowpass and highpass, or bandpass and bandstop, responses, depending on the filter type [8]–[9]. Although IIR

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filters have non-linear phase for WDFs that are derived from classical Butterworth and Chebyshev analogue filters, the phase response is almost linear throughout the passband. For applications that do not require perfect linear phase response, these filters are ideal since they are more efficient than FIR filters. IIR filters require fewer multiplying coefficients to implement than IIR filters for the same magnitude response specifications.

This paper will show the design of WDFs that are derived from passive ladder analogue pulse-shaping filters. The WDF networks are configured from established first order sections but arranged in a manner similar to analogue pulse-shaping networks. In addition, the coefficients and responses of the WDF are derived in terms of the digital equivalent of the analogue domain pulse-shaping parameters. The advantages of this design include obtaining highly modular structures, having direct correspondence between the design parameters in both the analogue and digital domains, and minimising the number of digital filter coefficients requirement. The WDFs obtained for CR-RC type and semi-Gaussian shaping are also tunable with a single multiplier coefficient for each of the modular components.

2. CR-RC Pulse-Shaping Network

Pulse-shaping is required to process a train of pulses produced by a preamplifier of a radiation detector. The pulses from the preamplifier tend to overlap one another due to the pulse-long duration and short, sharp peak. The term CR-RC shaping refers to the use of passive resistor capacitor networks to carry out a desired alteration in the pulse shape. Traditional CR-RC pulse-shaping, analogue networks are low-pass and high-pass filters also known as integrators and differentiators, respectively. The shaping shortens the pulse and changes its peak amplitude.

In a resistor capacitor filter network, the critical parameter is the time constant given by

$$\tau = RC \qquad \text{........(1)}$$

where R is the resistance and C, the capacitance of the circuit. The cut-off frequency of the filter is related to the time constant according to

$$\Omega_c = 1/\tau \qquad \text{........(2)}$$

For a basic *CR* differentiator network as shown in **Figure 1**, the equation that describes its response to a unit step input is

$$h(t) = e^{-t/\tau} u(t) \qquad \text{........(3)}$$

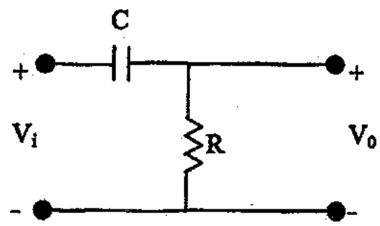


FIGURE 1: *CR Differentiator Network*

For a basic *RC* integrator network shown in **Figure 2,** the unit step response is

$$h(t) = (1 - e^{-t/\tau}) u(t) \qquad \text{........(4)}$$

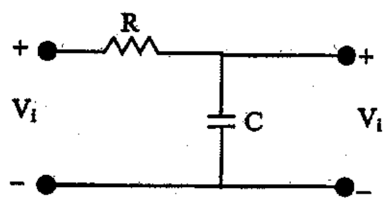


FIGURE 2: *RC Integrator Network*

A *CR-RC* network is obtained with the two networks above using a suitable impedance isolation network so that neither network influences the operation of the other [2]. An ideal isolation network is a unity gain amplifier with infinite input impedance and zero output impedance. For the *CR-RC* network shown in **Figure 3**, the unit step response is

$$h(t) = \frac{\tau_1}{\tau_1 - \tau_2}(e^{-t/\tau_1} - e^{-t/\tau_2}) u(t) \qquad \text{........(5)}$$

where τ_1 and τ_2 are the time constants of the differentiating and integrating networks, respectively. If the time constants are of equal value, τ, the unit step response is

$$h(t) = \left(\frac{t}{\tau}\right) e^{-t/\tau} u(t) \qquad \text{........(6)}$$

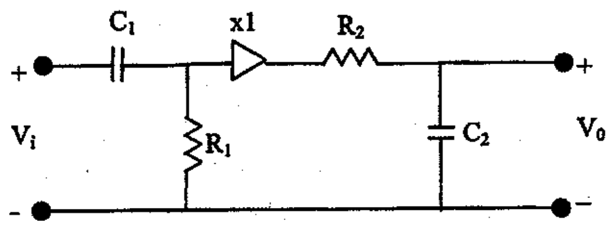


FIGURE 3: *CR-RC Shaping Network*

Varying unequal time constants changes the maximum amplitude of the output signal and the time that it occurs with respect to the relative values of the time constants. When both time constants are equal to τ, the maximum output amplitude is

$$h(t)_{\max} = e^{-1} \qquad \text{........(7)}$$

occurring at the instance, called the peak time,

$$\tau_{peak} = \tau \qquad \text{........(8)}$$

Basic *CR-RC* shaping produces non-symmetrical pulses. Better symmetry is achieved by cascading a single *CR* differentiation network with several stages of *RC* integrators, with the differentiator and integrator networks having equal time constants. This results in semi-Gaussian shaping. One of the advantages of the semi-Gaussian pulse is that the more symmetric shape results in a faster return to baseline, thereby reducing pulse pile-up [2].

For a differentiator followed by *p* sections of the *RC* integrator network, the step response is

$$h(t) = \left(\frac{t}{\tau}\right)^p \frac{1}{p!} e^{-t/\tau} u(t) \qquad \text{........(9)}$$

A more symmetrical shape is achieved at the expense of delayed peak time and reduced maximum amplitude. The time required for the shaped pulse to reach its maximum is

$$\tau_{peak} = p\tau \qquad \text{........(10)}$$

while the maximum amplitude is

$$h(t)\max = p^p \frac{1}{p!} e^{-p} \qquad \text{........(11)}$$

The time constants can be adjusted to give similar peak time for both the *p* lowpass section network and the *p=1* lowpass section. This can be achieved by scaling the time constants of all the sections of the *p* lowpass network, including the highpass section, according to

$$\tau_p = \tau_{p=1} / p \qquad \text{........(12)}$$

The choice for the time constant of the shaping networks depends on the charge collecting time of the detector used. The time constant is often selected to optimise the performance of the shaper by optimising the signal to noise ratio [1]. Thus, by making the time constant variable, an optimum performance can be achieved depending on the circumstance. For analogue filters, changing the time constant requires changing the analogue components according to (1). For digital filters, this is achieved by varying the coefficient values.

3. WDF Elements and Adaptors

The relationship between a WDF, operating at a sampling rate *F=1/T*, and its reference analogue filter is established in the frequency domain by the means of the bilinear transformation

$$s = \frac{2}{T}\left(\frac{z-1}{z+1}\right) \qquad \text{........(13)}$$

where *s* is the reference analogue domain and *z* the digital domain. The relationship between the analog frequency Ω on the *s* plane and the digital frequency ϖ on the *z* plane is

$$\Omega = \frac{2}{T}\tan(\varpi/2) \qquad \text{........(14)}$$

Direct realisation of the digital filter structure obtained with (13) will produce signal flow-graphs with delay-free loops. To overcome this, WDFs use wave network characterisation to obtain realisable digital structures.

Each element in the analogue domain is represented as a classical port characterised by a current, *i*, voltage, *v*, and port resistance, *R*. The signals going into a port is known as the incident wave, *a*, and the signal going out of the port is the reflected wave, *b*. The equations that relate these parameters are [8]

$$a = v + iR \qquad \text{........(15a)}$$

$$b = v - iR \qquad \text{........(15b)}$$

Each port is represented by the wave quantities and a port resistance. The incident and reflected waves are the signal variables of WDFs. The port resistance value depends on the analogue elements that it models and these parameters are shown in **Figure 4**.

To form a WDF structure, connections between WDF elements are required. This is accomplished using adaptors that connect the elements in accordance to Kirchoff's laws, thus making the structures realisable. The adaptors contain the multipliers and adders of the WDF. Adaptors are categorised according to the number of ports that they connect. In addition, adaptors are classified according to the type of connection that they realise, either series or parallel, with the exception of the two-port adaptor that realises both the series and parallel connections. The most common adaptors are the two-port adaptor, three-port series and parallel adaptors.

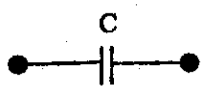
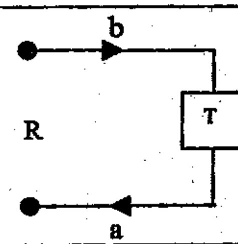
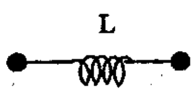
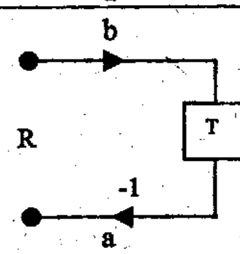
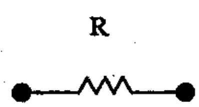
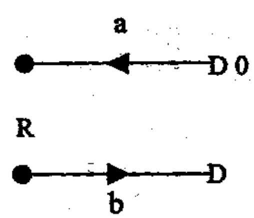
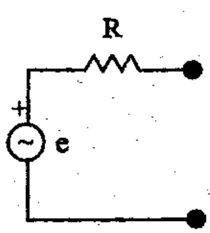
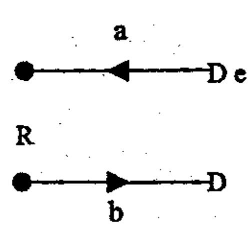
ELEMENT	PORT RESISTANCE (R)	WDF EQUIVALENT
 Capacitor	T/2C	
 Inductor	2L/T	
 Resistor	R	
 Resistive source	R	

FIGURE 4: *WDF Equivalence of Analogue Elements*

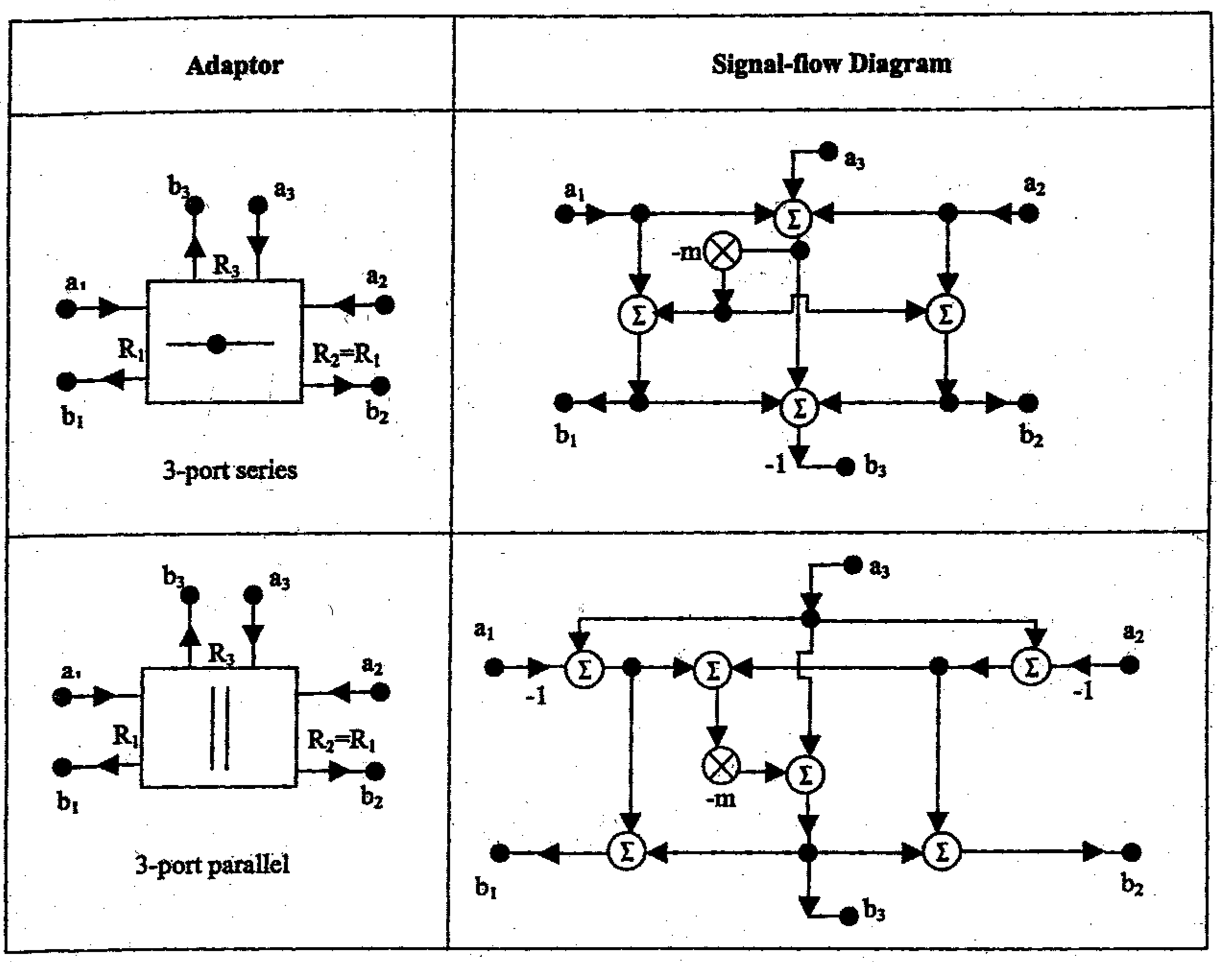


FIGURE 5: *WDF 3-Port Adaptors with Single Multiplier*

The WDFs discussed in this paper use three-port series and parallel adaptors realised in an efficient manner requiring only a single multiplier per adaptor. This is obtained when two of the ports are equal in value. These adaptors along with their signal flow graphs are shown in **Figure 5**.

The equations that describe the three-port series adaptor, with $R_1 = R_2$, are [8]

$$b_1 = a_1 - m(a_1 + a_2 + a_3) \qquad \text{........(16a)}$$

$$b_2 = a_2 - m(a_1 + a_2 + a_3) \qquad \text{........(16b)}$$

$$b_3 = -(b_1 + b_2 + a_1 + a_2 + a_3) \qquad \text{........(16c)}$$

where the coefficient of the adaptor is

$$m = \frac{2R_1}{2R_1 + R_3} \qquad \text{........(17)}$$

The equations that describe the three-port parallel adaptor are

$$b_1 = b_3 + (a_3 - a_1) \qquad \text{........(18a)}$$

$$b_2 = b_3 + (a_3 - a_2) \qquad \text{........(18b)}$$

$$b_3 = a_3 - m(a_3 - a_1) - m(a_3 - a_2) \qquad \text{........(18c)}$$

where

$$m = \frac{2G_1}{2G_1 + G_3} \qquad \text{........(19)}$$

with G being the conductance of the port; $G = 1/R$.

4. Pulse-Shaping WDF Networks

4.1 Equivalent Filter Networks

The basic differentiator network shown in **Figure 1** and the integrator network shown in **Figure 2** can be realised with equivalent WDF networks. An efficient realisation is obtained by using lossless analogue filters terminated by equal value resistive source and load. These filters can be designed to have the same function as the pulse-shaping networks. The analogue network and its WDF equivalent is shown in **Figure 6.**

The WDF network is characterised by the scattering matrix [8]

$$S = \begin{vmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{vmatrix} \qquad ....(20)$$

where

$$s_{11} = \frac{a_1}{b_{1/a_2=0}} \qquad ....(21a)$$

$$s_{12} = \frac{a_1}{b_{2/a_2=0}} \qquad ....(21b)$$

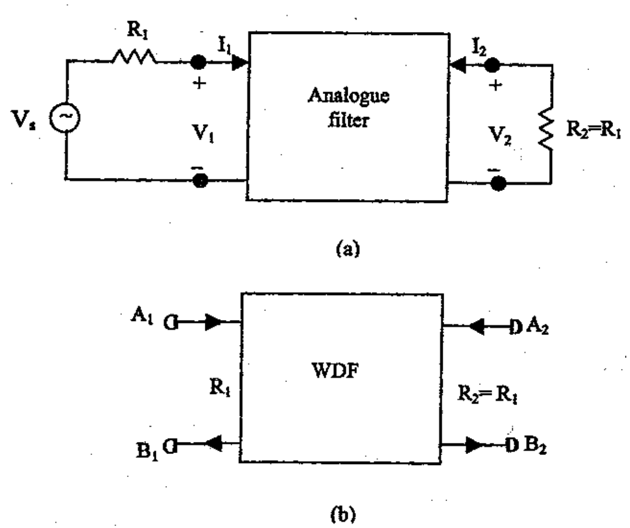


FIGURE 6: *(a) Analogue Network and (b) WDF Equivalent Network*

$$S_{21} = \frac{a_2}{b_{1/a_1=0}} \qquad ....(21c)$$

$$s_{22} = \frac{a_2}{b_{2/a_1=0}} \qquad ....(21d)$$

For WDFs derived from lossless analogue networks,

$$\mathbf{S}^T \mathbf{S}^* = \mathbf{E} \qquad ....(22)$$

where **E** is the unit matrix. This results in a WDF network having a lowpass transfer function, S_{21}, and a highpass transfer function, S_{11}, which are power complementary, where

$$|S_{21}|^2 + S_{11}|^2 = 1 \qquad ....(23)$$

The WDF can realise both the highpass and lowpass responses using the same structure.

A first order Butterworth filter is shown in **Figure 7**. Although other types of analogue filters can be used, such as Chebyshev or Elliptic, Butterworth filters have the advantage of having maximally-flat response in the passband. The values of the analogue components normalised to the cut-off frequency 1 rad/sec are $R_1 = 1\,\Omega$ and L = 2H. To change the cut-off frequency to a frequency Ωc requires scaling the inductance according to

$$L = \frac{2}{\Omega_c} \qquad ....(24)$$

To connect the WDF equivalents of these elements in the ladder configuration, a three-port series adaptor is required. Converting these values to the port resistance

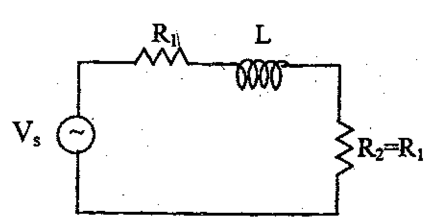


FIGURE 7: *First Order Butterworth Filter*

values, with ports 1 and 2 representing the resistive terminations and port 3 as the inductor, and from (17) results in

$$m=\left(\frac{\Omega_c T}{\Omega_c T+2}\right) \qquad \text{........(25a)}$$

For pulse-shaping networks considered in this paper, the coefficient can be written in terms of the time constant as

$$m=\left(\frac{T}{T+2\tau}\right) \qquad \text{........(25b)}$$

The coefficient of the adaptor is related to the cut-off frequency and therefore the time constant of the analogue network. For the purpose of digital, pulse-shaping network, the coefficient, however, is more conveniently expressed in terms of the digital cut-off frequency or time constant. From (14) and (25), the coefficient is

$$m=\frac{\tan(\omega_c/2)}{\tan(\omega_c/2)+1} \qquad \text{........(26a)}$$

or

$$m=\frac{\tan(1/2\tau_D)}{\tan(1/2\tau_D)+1} \qquad \text{........(26b)}$$

where the τ_D is defined as the digital time constant of the WDF with

$$\tau_D=\frac{1}{\omega_c} \qquad \text{........(27)}$$

The series adaptor realisation of the first order WDF is shown in **Figure 8.** The input signal is fed into the first adaptor, while the second port input is set to zero. The WDF has the highpass and lowpass responses as the output signal of the first and second port, respectively, while the third port represents the inductor.

An alternative WDF realisation using a parallel adaptor is obtained from a first order network consisting of a capacitor connected in parallel with the resistors. This is the dual of the network shown in **Figure 7**. However, since both types of networks realise the same transfer function, it can be shown that the resulting WDF has the same transfer function and coefficient value.

4.2 CR-RC WDF Pulse-Shaping Network

To realise *CR-RC*-type-shaping using WDFs, a cascade of two first order networks are required, representing the differentiator and integrator sections. The connection between the first section performing the high-pass filtering operation and the second section performing the lowpass filtering operation is shown in **Figure 9** for series adaptors.

The step response of the WDF section is the sampled version of its analogue counterpart. Using the digital time constant as defined in (27), the highpass WDF response to a step input is

$$h(n)=e^{-n/\tau_D}u(n) \qquad \text{........(28)}$$

where n is the time index of the sampled signal.

The lowpass WDF step response is

$$h(n)=(1-e^{-n/\tau_D})u(n) \qquad \text{........(29)}$$

The step response for the resulting WDF network shown in **Figure 9** is

$$h(n)=\frac{\tau_{D1}}{\tau_{D1}-\tau_{D2}}(e^{-n/\tau_{D1}}-e^{-n/\tau_{D2}})u(n) \qquad \text{........(30)}$$

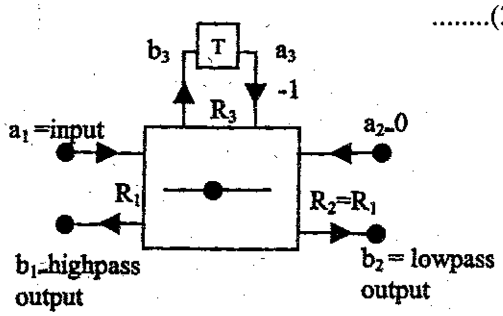


FIGURE 8: *First Ordeer WDF with Series Adaptor*

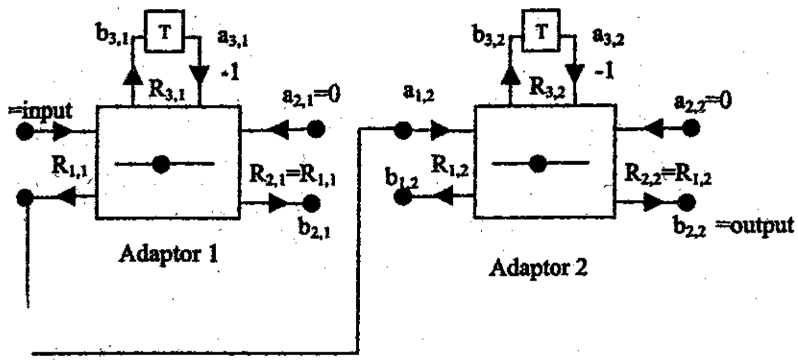


FIGURE 9: *Highpass and Lowpass WDF Shaping Network*

where τ_{D1} and τ_{D2} are the digital time constants of the highpass and lowpass WDFs, respectively.

If the time constant for both the lowpass and highpass networks is equal to τ_D the step response is

$$h(n) = \left(\frac{n}{\tau_D}\right) e^{-n/\tau_D} u(n) \qquad ....(31)$$

The response of the WDF is dependent on the digital time constant and hence multiplying coefficient, *m*. The effects of varying *m* for the WDF networks are shown in **Figure 10** for different values of *m* for the highpass and lowpass sections. Similar to the analogue

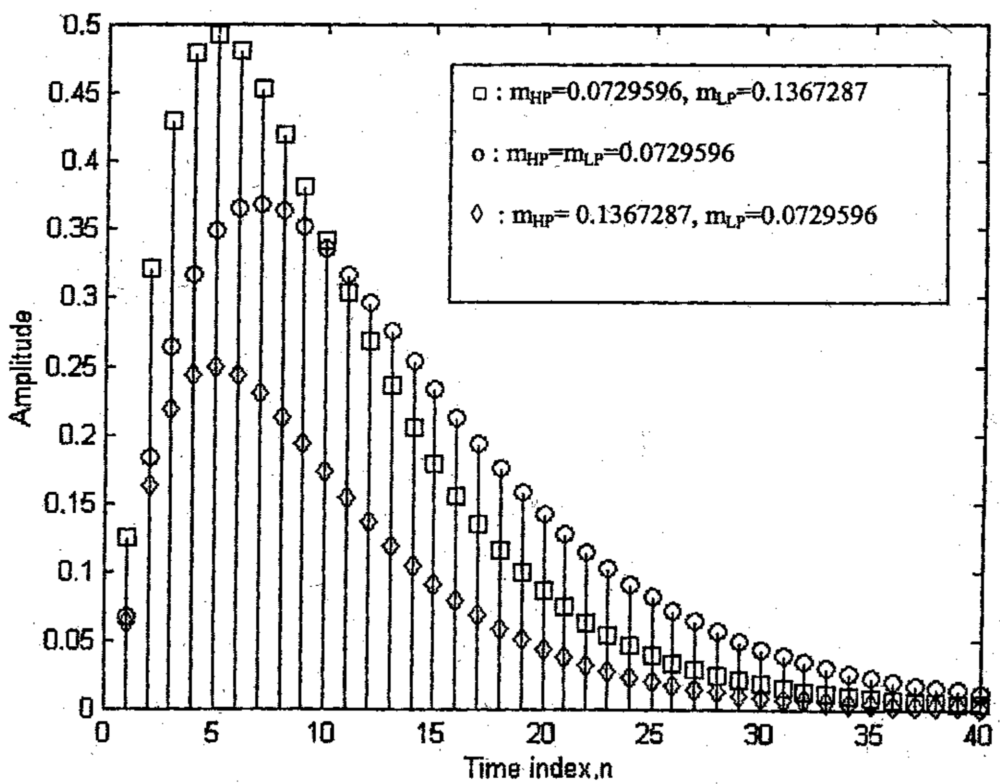


FIGURE 10: *Varying the Coefficients of the WDF Highpass (m_{HP}) and Lowpass (m_{LP}) Sections shifts the Maximum Amplitude and Peak Time*

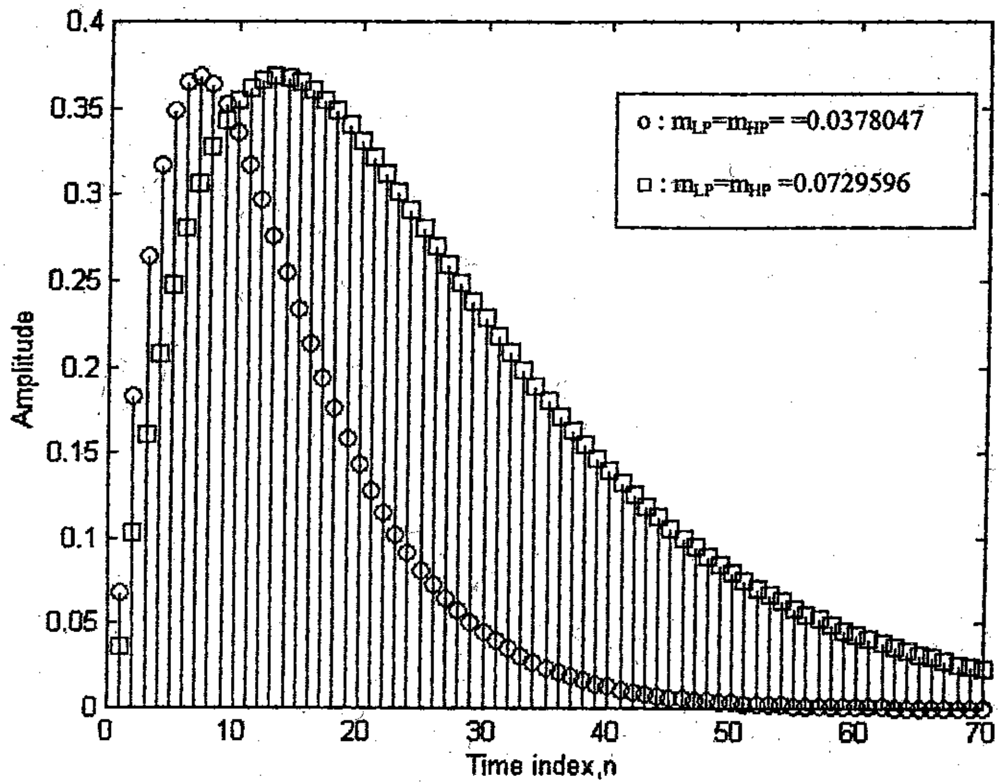


FIGURE 11: *Varying the Coefficients of Equal Value Lowpass and Highpass Sections shifts the Peak Time while maintaining the Maximum Amplitude*

network, the peak time and maximum amplitude of the output pulse depends on the relative values of the time constants.

Figure 11 shows the effects of varying the coefficient on the output pulses for two different cases with the lowpass and highpass sections having the same multiplying coefficient value. The maximum amplitude is as given in (7) while the peak time varies according to the digital time constant

$$\tau_{peak} = \tau_D \qquad \text{........(32)}$$

This peak will only coincide with a sampling index n if the cut-off frequency is selected such as (27) is an integer. Otherwise, the peak will occur in between two sampling instances.

Several configurations for the WDF network are possible. The order of the lowpass and highpass sections can be interchanged due to the linearity property of digital filters. An equivalent realisation with three-port parallel adaptors will produce similar shaping. The adaptors can also be of different types, series in combination with parallel, giving a more complex realisation. Another alternative realisation of the WDF network is by using resonant second order circuit as the reference network. The resulting WDF will have two different coefficients that control the bandwidth and centre frequency [8]. However, simple cascading of equivalent lowpass sections to obtain semi-Gaussian shaping is not realisable with second order WDF sections.

4.3 Semi-Gaussian WDF Pulse-Shaping Networks

To realise semi-Gaussian shaping, a cascade of the lowpass sections is required following the highpass section. This configuration is shown in **Figure 12.** For p lowpass sections, the step response is

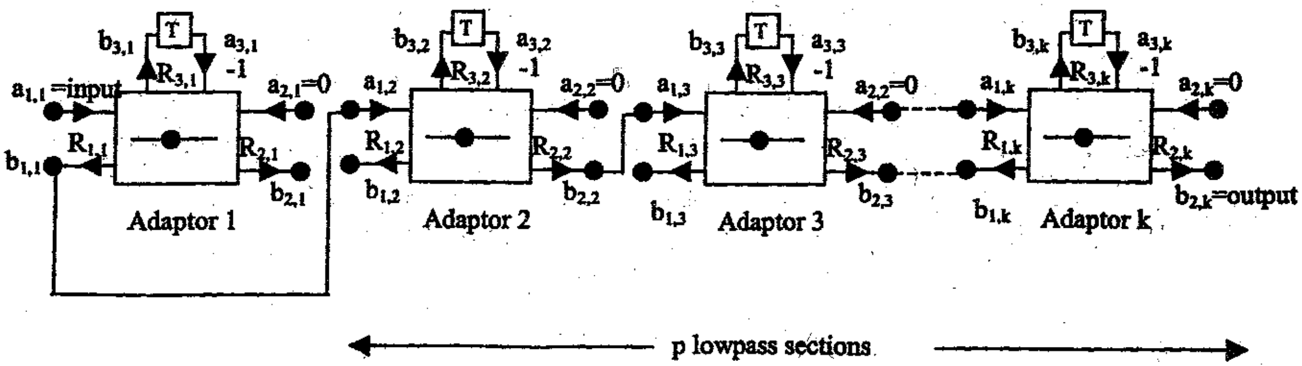


FIGURE 12: *Semi-Gaussian Pulse-Shaping WDF Network*

$$h(n)=\left(\frac{n}{\tau_D}\right)^p \frac{1}{p!}e^{-n/\tau_D}u(n) \qquad ....(33)$$

As with the analogue network, increasing p has the effect of decreasing the maximum amplitude according to (11). The peak time is

$$\tau_{peak}=p\tau_D \qquad ....(34)$$

The effect of increasing p on the peak time and maximum amplitude is shown in **Figure 13** with the same coefficient value for all sections. As with the lowpass and highpass shaping network, the peak will only coincide with a sampling index n if the cut-off frequency is selected such as (27) is an integer.

The delay of the p lowpass sections network can be shortened to be equivalent to a delay τ_D obtained using a single low-pass section by scaling all the time constants according to

$$\tau_{D_p}=\tau_{D_{p=1}}/p \qquad ....(35)$$

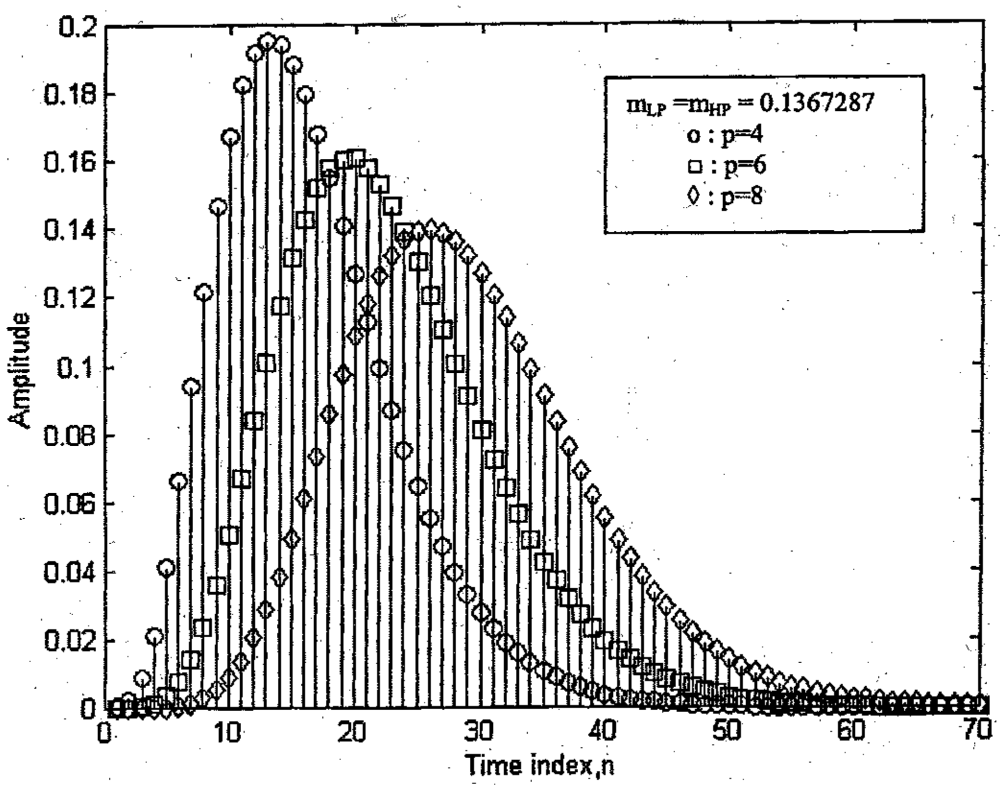


FIGURE 13: *Semi-Gaussian Shaping is obtained using p WDF Lowpass Sections with the Same Coefficient for All Sections*

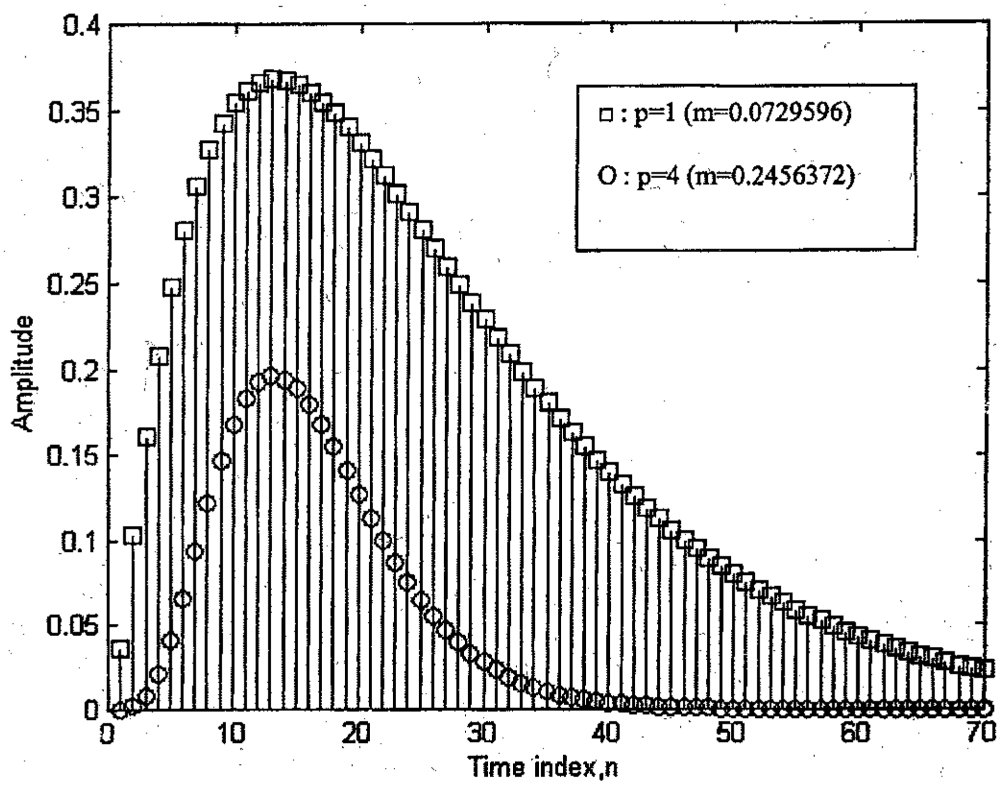


FIGURE 14: *Semi-Gaussian Shaping with Scaled Coefficients shifts the Peak Time of p = 4 Lowpass WDF Sections to equal that obtained with p = 1 Lowpass Section*

This is equivalent to scaling the cut-off frequency of the p lowpass sections WDF network and the highpass section according to

$$\varpi_{c_p} = p\varpi_{c_{p=1}} \qquad \text{........(36)}$$

The effects of changing the cut-off frequency by tuning the coefficient is shown in **Figure 14** comparing $p = 4$ and $p = 1$ WDF networks having the same peak time.

5. Coefficient Comparison

Conventional IIR filters can be used to realise digital pulse-shaping networks using a cascade of first order sections. For an N order filter, the number of coefficients required for a canonic realisation with respect to the number of delays is 2N+1. If an FIR filter is used, more coefficients are required to meet the same response as an IIR filter [10]. Hence, for a first order filter, at least three coefficients are required.

The WDF realisation thus requires 1/3 the number of coefficients for each filter section when compared to conventional IIR filters. The advantage increases when several lowpass sections are used to obtain the semi-Gaussian shape. In addition, the IIR and FIR filters require different coefficient values and structures to realise highpass and lowpass filters. This is unlike WDFs where the same adaptor is used to realise both the lowpass and highpass sections.

With all the digital filters, an additional coefficient can be added to the output of the filter structure to scale the amplitude of the output pulses. The value of the coefficient can be set to the reciprocal of the maximum amplitude value for the pulses as given in (7) and (11) for the case of CR-RC type and semi-Gaussian shaping, respectively.

6. Conclusion

This paper has shown the design of pulse-shaping WDF networks. The correspondence between the multiplier coefficient and the analogue and digital domain time constants are shown along with the output responses. By arranging the first order WDF sections according to analogue filter networks, CR-RC type and semi-Gaussian shaping are obtained. The WDF networks are highly modular as the same type of adaptor is used to realise the highpass and lowpass responses. A minimum number of coefficients is required when compared to other types of digital filter networks. Each adaptor has a single multiplier that is tunable to vary both the peak time and amplitude of the output pulses. A scaling coefficient can be used at the output of the pulse-shaping WDF networks to adjust the amplitude altered by the digital filtering operations.

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