Design of Controllers for Microstructure Development

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In this paper, a new strategy for systematically calculating optimal process parameters for microstructural control during hot extrusion process is presented. Modem control theory has been applied to the microstructure development during hot extrusion. In the present study, an attempt has been made to control the microstructure evolution during extrusion using the conventional PI Control, Generalised Predictive Control (GPC) and Linear Quadratic Regulator (LQR) approaches. This approach treats the deforming material as a dynamical system and involves developing state space models from available material behaviour and hot deformation process models. The final grain size and volume fraction of recrystallisation after extrusion are considered as the optimal criterion and the grain size is expressed in terms of strain, strain rate and temperature. The steps involved in conventional PI Control and GPC approach include process modelling and controller design. LQR approach is based on optimal control theory and involves developing of state space models to describe the material behaviour. The trajectories of the independent variables to achieve the desired grain size are obtained and the strain values are further utilised to optimise the dimensions of the extrusion die profile to achieve the required grain size. Also, the performance of PI, GP controllers and LQR are compared through ISE and IAE values. Simulation studies are carried out using MATLAB software.

Keywords: Microstructure control, extrusion, PI control, Generalised Predictive Control and Linear Quadratic Regulator.

1. Introduction

The development of optimal design and control methods for extrusion process is needed for effectively reducing the part cost, reduction in sizes, improving part delivery schedules and producing specified part quality on a repeatable basis. Existing design methods generally lack adequate capabilities for finding effective process parameters such as deformation rate, die and work piece temperature and tooling system configuration.

This situation presents major challenges to control engineers who are faced with higher yield requirements and superior quality standards. Therefore, it is important to develop new systematic methodologies for process design and control based upon scientific principles, which sufficiently consider the behaviour of work piece material and the mechanics of the manufacturing process (Venugopal & Frazier, 1997).

Extrusion is a process by which the cross section of a billet is reduced by forcing it to flow through the die. The process is used to refine the larger grain size to smaller grain size. The refinement of grains during extrusion is influenced by several factors

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such as strain, strain rate, and temperature. Hot extrusion is usually performed by using converging dies, in which the cross-section of the die orifice changes gradually from the initial billet shape to the final product shape over the length of the die. The development of dynamic process models allows the user to systematically design a control system without resorting to *ad hoc* tuning methods. Therefore, the application of conventional PI control, GPC and LQR approaches have become attractive to achieve the desired objectives.

2. Objectives of the Present Study

The objectives of the present work are as follows:

- To develop a dynamic model for hot extrusion of 0.3% carbon steel.
- To determine the transfer function from the dynamic models for grain size and volume fraction of recrystallisation.
- To optimise the microstructural variables strain, strain rate and temperature and in turn the die profile and optimum ram velocity parameters for extrusion are to be optimised.
- To design PI, GP controllers and LQR and implement the same through simulation for the above-mentioned process.
- To obtain the closed loop responses for grain size and volume fraction of recrystallisation.
- To compare the performance of PI and GP controllers and LQR approach and by evaluating the criteria like ISE and IAE.

3. Problem Description

The control of microstructure in metal-working processes is needed for better quality. A methodology for computation of optimal control parameters during hot deformation process for microstructural control is proposed. This approach is based on designing and implementation of conventional PI and Generalised Predictive Controllers and LQR through simulation. It involves developing state space models from available material behaviour and hot deformation process models. The control system design consists of two stages – analysis and optimisation. In the analysis stage, using the empirical models for microstructure development, the optimum strain ($\varepsilon(t)$), strain-rate ($\dot{\varepsilon}(t)$), and temperature ($T(t)$) trajectories for processing are estimated. The available simulation models for ram velocity and extrusion profile are then used to calculate process control parameters such as ram velocity and die profile so as to achieve the strain, strain rate and temperature trajectories obtained earlier. The two-stage approach for microstructural control was applied to the hot extrusion of 0.3% carbon steel based on the models used by **Frazier *et al.*** (1998).

Average recrystallised grain size is given by

$$D = 22600(\dot{\varepsilon})^{-0.27} \; e^{-0.27(Q/RT)} \qquad \text{........(1)}$$

where

Activation energy, $Q = 267 \; KJ/Mol,$
Gas constant, $R = 8.314 \; X 10^{-3} \; KJ/Mol.k.$

Time derivative of average recrystallised grain size is given by

$$\dot{D} = \frac{d(D)}{dt} = \frac{d(D)}{dT} x \frac{dT}{dt} \qquad \text{........(2)}$$

$$\dot{D} = 0.27D \frac{Q}{RT^2} x \frac{dT}{dt} \qquad \text{........(3)}$$

The volume fraction of recrystallized grains is given by:

$$\chi = 1 - \exp\{ln(2)\,[(\varepsilon - \varepsilon_c)/\varepsilon_{0.5}]^2\} \qquad \text{........(4)}$$

where

ε - strain

ε_c - critical strain and

$\varepsilon_{0.5}$ - strain for 50% volume recrystallisation

Time derivative of volume fraction of recrystallised grains is given by:

$$\dot{\chi} = \frac{2 \ln 2}{(\varepsilon_{0.5})^2}(\varepsilon - \varepsilon_c)(1-\chi)\dot{\varepsilon} \quad \text{........(5)}$$

Time derivative of temperature is given by

$$\dot{T} = \frac{\eta}{\rho C_p}\sigma(\varepsilon, \dot{\varepsilon}, T)\dot{\varepsilon} \quad \text{........(6)}$$

where

σ - flow stress (kPa)

C_p - Specific heat capacity (381 J/Kg K)

ρ - Density (7.8 kg/m^3)

η - Fraction of work, which transforms into heat (0.95).

$$\sigma = \sinh^{-1}[(\dot{\varepsilon}/A)^{1/n}\, e^{Q/nRT}]/0.0115\, X\, 10^{-3} \quad \text{........(7)}$$

$$\ln A(\varepsilon) = 13.92 + 9.023/\varepsilon^{0.502} \quad \text{........(8)}$$

$$n(\varepsilon) = -9.07 + 3.787/\varepsilon^{0.368} \quad \text{........(9)}$$

$$Q(\varepsilon) = 125 + 133.3/\varepsilon^{0.393} \quad \text{........(10)}$$

4. Optimisation of Process Parameters

The trajectories of strain, strain rate and temperature are obtained in the first stage. Process parameters such as die geometry, ram velocity and billet temperatures are obtained in the second stage. The ram velocity V_{ram} is given by

$$V_{ram} = \frac{L}{\int_0^t \varepsilon(t)dt} \quad \text{........(11)}$$

where L = die length, $\varepsilon(t)$ = strain trajectory.

The die shape can be described by the radius r and axial distance (die throat length) y, where $r(t) = r_0 e^{-\varepsilon(t)/2}$, $y(t) = V_{ram}\int_0^t e^{\varepsilon(t)}dt.$

r_0 - radius at entrance and The optimal die profile for achieving final grain size of 26 μm is obtained using the above methodology.

5. Conventional Control System

The function of a controller is to receive the measured process variable (pv) and to compare it with the set point (sp) so as to produce the actuating signal (m) that drives the process variable to the desired value. Thus, the input to the controller is the error $(sp - pv)$

Depending on the relation between the error and the controller output signal, controllers are classified as Proportional, Proportional + Integral (PI), Proportional + Integral +Derivative (PID) controllers (Stephanopoulos, 2001).

5.1 Controller Design

5.1.1 PI Controller

PI controller is also known as proportional plus reset controller. The actuating signal $m(t)$ is related to the error $e(t)$ by the equation

$$m(t) = K_c\, e(t) + \frac{K_c}{T_i}\int_0^t e(t)dt + m_s \quad \text{........(12)}$$

where K_c is the controller gain, m_s is the bias, T_i is the integral time constant or reset time and $1/T_i$ is the repeats per minute. After a period of T_i minutes, for a constant error E, the contribution of integral term is

$$\frac{K_c}{T_i}\int_0^{T_i} e(t)dt = \left(\frac{K_c}{T_i}\right)ET_i = K_c E \quad \text{........(13)}$$

The integral action has repeated the response of the proportional action. Reset time is the time needed to repeat change in output due to initial proportional action. The integral action causes the controller output $m(t)$ to change as long as an error exists in the process output.

The Transfer function of a PI Controller is

$$G_c(s) = K_c\left[1+\frac{1}{T_i s}\right] \quad ....(14)$$

5.1.2 Controller-Tuning

Determination of the controller parameters to provide the desired response is known as controller-tuning. Controller-tuning can be done by Cohen-Coon open loop method.

5.1.3 Cohen-Coon Method

This method is an open loop method in which the controller action is removed from the process and an open loop transient is induced by a step change in the signal. The step response of the system is called the

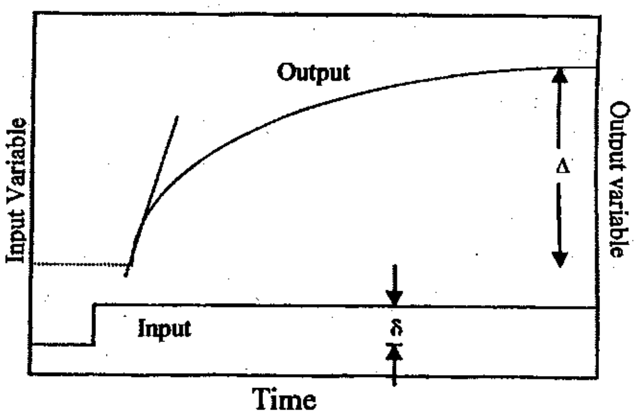

FIGURE 1: *Process Reaction Curve*

process reaction curve that exhibits an S-shaped curve as shown in **Figure 1**.

The process reaction curve method is the most widely-used technique for identifying the dynamic model of the process. This method provides adequate models for many applications. In this process reaction curve method, a step signal (a steady $d.c$ input) is given to the process and its output is allowed to reach steady state. A curve is obtained as shown in **Figure 1**.

From the process reaction curve, the process model is determined as a first order process with dead time.

The process reaction curve is restricted to this model. The form of the model is given in Eq. (15). where $X(s)$ denotes the input and $Y(s)$ the output, both expressed in deviation variables.

$$\frac{Y(s)}{X(s)} = \frac{Kpe^{-td.s}}{\tau s+1} \quad ....(15)$$

where Kp is the process gain, t_d is the time delay and τ is the time constant. The intermediate values determined from the graph are the magnitude of the input change δ; the magnitude of the steady-state change in the output Δ and the times at which the output reaches 28.3% and 63.2% of its final value. Any two values of time can be selected to determine the unknown parameters t_d and τ.

The typical times are selected where the transient response is changing rapidly so that the model parameters can be accurately determined in spite of measurement noise. The expressions are given by

$$Y(t_d+\tau) = \Delta(1-e^{-1}) = 0.632\Delta \quad ....(16)$$

$$Y(t_d+\frac{\tau}{3}) = \Delta(1-e^{-1/3}) = 0.283\Delta \quad ....(17)$$

Thus, the values of time at which the output reaches 28.3% and 63.2% of its final value are used to calculate the model parameters [4].

$$t_{28.3\%} = t_d + \tau/3 = t_1 \quad ....(18)$$

$$t_{63.2\%} = t_d + \tau = t_2 \quad ....(19)$$

$$\tau = 1.5(t_2 - t_1) \quad ....(20)$$

$$T_d = t_2 - \tau \quad ....(21)$$

$$K_p = \Delta/\delta \quad ....(22)$$

After the calculation of K_p and t_d, the controller-tuning parameters K_c, T_i are calculated using the Cohen-Coon formulae.

For PI controller, the controller parameters are expressed as

$$K_c = \frac{1}{K_p} x \frac{\tau}{t_d}\left(0.9 + \frac{t_d}{12\tau}\right) \quad \text{........(23)}$$

$$T_i = t_d \; x \left[\frac{30 + 3\frac{t_d}{\tau}}{9 + 20\frac{t_d}{\tau}}\right] \quad \text{........(24)}$$

5.1.4 Synthesis Method

Design of controller by synthesis method is one of the recommended methods in process control. However, the synthesis method is applicable only for open loop stable process in order to get a stable controller. Hence, the unstable process must be stabilised using a proportional controller (Chidambaram, 1998).

The synthesis controller is the combination of two parts. The first part compensates for the process transfer function and the other is used to obtain a specified closed loop response of the controlled variable to the set point. When the process transfer function does not have dead time, the closed loop transfer function becomes

$$\frac{Y(s)}{X(s)} = \frac{Kp}{\tau_c s + 1} \quad \text{........(25)}$$

The closed loop time constant τ_c can be adjusted to shape the response of the loop, the smaller the τ_c, the faster the controller response. Thus, τ_c provides a convenient parameter to reach a compromise between fast approach to set point and acceptable variations in the controller output.

The PI controller parameters are

$$Kc = \tau / Kp\,\tau_c \quad \text{........(26)}$$

$$T_I = \tau \quad \text{........(27)}$$

In this problem, the desired output of the system must possess the required grain size and volume fraction of recrystallisation. The block diagram of conventional PI control system is shown in **Figure 2.**

6. Generalised Predictive Controller

Current self-tuning algorithms lack robustness to prior choices of either dead-time or model order.

A novel method GPC is also designed and implemented through simulation for the above process. This receding-horizon method depends on predicting the plant output over several steps based on assumptions about future control actions.

Although self-tuning and adaptive control have made much progress over the past decade, both in terms of theoretical understanding and practical applications, none of the methods proposed so far is best suited as a "general purpose" algorithm for the control of the majority of real processes. It is capable of stable control of processes with parameters that change instantaneously, provided that the input-output data is sufficiently rich to allow reasonable plant identification. Also, GPC is best suited for processes with variable dead time and with a model order. Also,

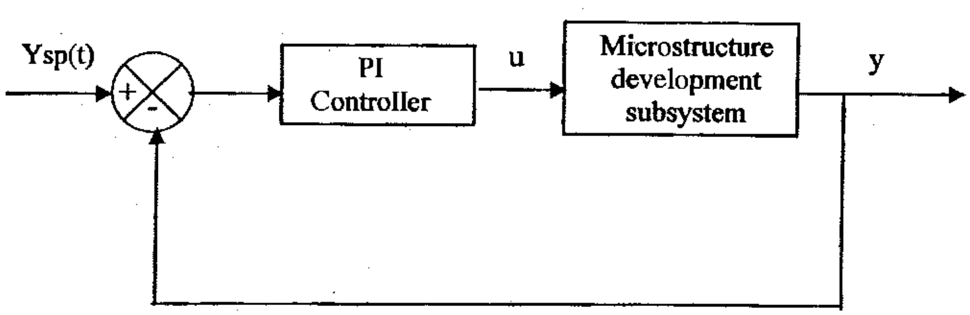

FIGURE 2: *Block Diagram of Conventional PI Control System*

it is best suited for high-performance applications such as the control of flexible systems. The GPC approach being based on an explicit plant formulation can deal with variable dead time but as it is a predictive method, it can also cope with over-parameterisation (Clarke et al, 1987).

6.1 The CARIMA Plant Model and Output Prediction

When the regulation about a particular operating point is considered, even a non-linear plant generally admits a locally-linearised model:

$$A(q^{-1})y(t)=B(q^{-1})u(t-1)+x(t) \quad \text{........(28)}$$

where A and B are polynomials in the backward shift operator q^{-1} expressed as:

$$A(q^{-1})=1+a_1q^{-1}+.....+a_{na}q^{-na}$$

$$B(q^{-1})=b_0+b_1q^{-1}+.....+b_{na}q^{-nb}$$

If the plant has non-zero dead-time, the leading elements of the polynomial $B(q^{-1})$ are zeroes. In Eq. (28), $u(t)$ is the control input, $y(t)$ is the measured variable or output, and $x(t)$ is the disturbance input.

In the literature, $x(t)$ has been considered to be moving average form:

$$x(t)=C(q^{-1})\xi(t) \quad \text{........(29)}$$

where $C(q^{-1})=1+c_1q^{-1}+.....+c_{nc}q^{-nc}$

In this equation, $\xi(t)$ is an uncorrelated random sequence, and combining with Eq. (28) CARMA (Controlled Auto-Regressive and Moving-Average) model is obtained.

$$A(q^{-1})y(t)=B(q^{-1})u(t-1)+C(q^{-1})\xi(t) \quad \text{........(30)}$$

In practice, two principal disturbances are encountered: random steps occurring at random times (e.g., change in material quality) and Brownian motion (found in plants relying on energy balance). In both these cases, an appropriate model is:

$$x(t)=C(q^{-1})\xi(t)/\Delta \quad \text{........(31)}$$

where Δ is the differencing operator $1-q^{-1}$. Coupled with (28), this gives the CARIMA model (integrated moving-average):

$$A(q^{-1})y(t)=B(q^{-1})u(t-1)+C(q^{-1})\xi(t)/\Delta \quad \text{........(32)}$$

For simplicity in the development here, $C(q^{-1})$ is chosen to be 1 and the resulting model is

$$A(q^{-1})y(t)=B(q^{-1})u(t-1)+\xi(t)/\Delta \quad \text{........(33)}$$

The identity

$$1=Ej(q^{-1})A\,\Delta+q^{-j}Fj(q^{-1}) \quad \text{........(34)}$$

is considered for deriving a j- step ahead predictor of $y(t+j)$ based on Eq. (33). Where Ej and Fj are polynomials uniquely defined given $A(q^{-1})$ and the prediction interval j. Multiplying Eq. (33) by $Ej\Delta q^{j}$,

$$EjA\Delta y(t+j)=EjB\Delta u(t+j-1)+Ej\xi(t+j) \quad \text{........(35)}$$

and substituting for $EjA\Delta$ from Eq. (29) gives:

$$y(t+j)=EjB\,\Delta u(t+j-1)+$$

$$Fj\,y(t)+Ej\xi(t+j) \quad \text{........(36)}$$

As $Ej(q^{-1})$ is of degree $j-1$, the noise components are all in the future time, so that the optimal predictor, given measured output data up to time t and any given $u(t+i)$ for $i>1$, is clearly:

$$\hat{y}(t+j/t) = Gj\,\Delta u(t+j-1) + Fj\,y(t) \quad \text{........(37)}$$

where $Gj(q^{-1}) = EjB$.

$$Gj(q^{-1}) = B(q^{-1})[1-q^{-j}Fj(q^{-1})]/A(q^{-1})\Delta.$$

Which means that one way of computing Gj is simply to consider the Z- transform of the plant's step response and to take the first j terms.

The GPC control law is obtained by minimising the following cost function:

$$J(N_1,N_2) = E\left\{\sum_{j=N_1}^{N_2}[y(t+j)-w(t+j)]^2 + \sum_{j=1}^{N_2}\lambda(j)[\Delta u(t+j-1)]^2\right\} \quad \text{........(38)}$$

where

$y(t+j)$	-	The future plant output at time $t+j$ based on the available input/output data at time t.
$w(t)$	-	The set-point.
N_1	-	Minimum prediction horizon.
N_2	-	Maximum prediction horizon.
N_u	-	Control horizon.
$\lambda \geq 0$	-	Control increment weighting.

It is worth that N_1, N_2, N_u and λ are the design parameters of GPC.

The minimisation of J assuming no constraints on future controls results in the projected control increment vector:

$$\tilde{u} = (G^TG+\lambda I)^{-1}G^T(w-f) \quad \text{........(39)}$$

The first element of $\tilde{u}$ is $\Delta u(t)$ so that the control input $u(t)$ is given by:

$$u(t) = u(t-1) + g^T(w-f) \quad \text{........(40)}$$

where g^T is the first row of $(G^TG+\lambda I)^{-1}G^T$.
$w = [w(t+1), w(t+2), \ldots\ldots w(t+N)]^T$
$f = [f(t+1), f(t+2), \ldots\ldots f(t+N)]^T$ and I is the unit matrix. The elements of the matrix G are the step response values and f is the free response (the response to an input fixed at $u(t-1)$ of the open loop system $B(q^{-1})/A(q^{-1})$.

7. Linear Quadratic Regulator

The LQR design and analysis involves linearising the nonlinear equations which describe the material behaviour and developing the state space model. The state space model together with an optimality criterion is used to control the grain size and percentage of volume fraction recrystallisation. The design

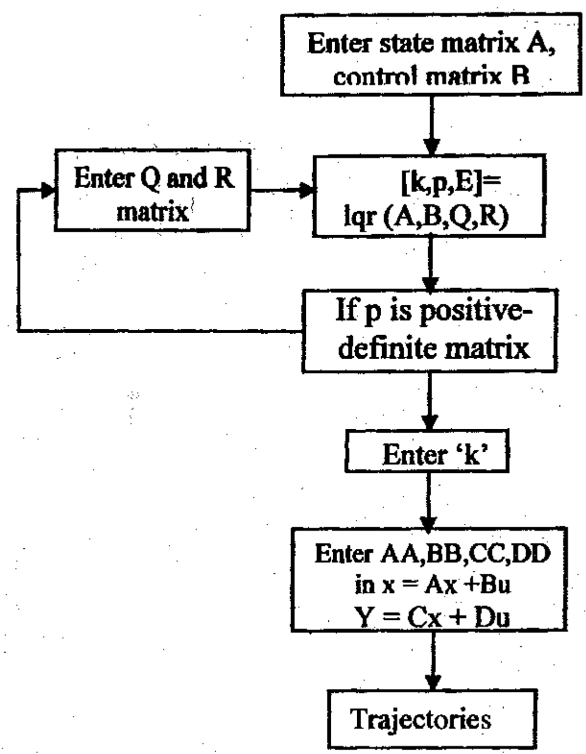

FIGURE 3: *Flow Chart for LQR*

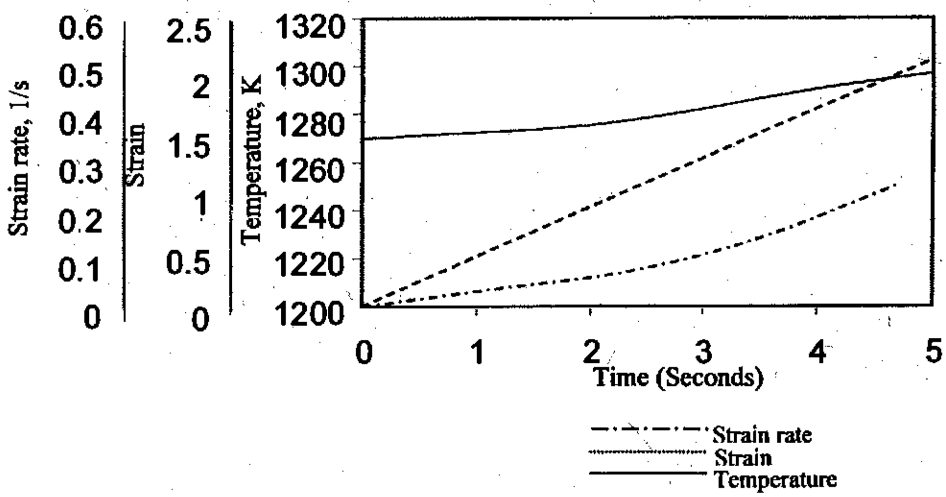

FIGURE 4: *Optimal Trajectories of Strain, Strain Rate and Temperature*

procedure for LQR is shown in **Figure 3** (Kirk, 1970).

Where,

k - gain matrix
p - matrix solution to riccati equation,
AA - state matrix
Q - positive definite matrix
R - real symmetric positive definite matrix
BB - control matrix
CC - output matrix
DD - transmission matrix

8. Optimising the Microstructural Trajectories and Extrusion Profile

The optimality criterion is chosen so as to attain a final strain of 2 while the recrystallised grain size is kept at a desired value of $26\mu m$ and the volume fraction of recrystallisation is 0.5. The average grain size of raw stock prior to extrusion is $180\mu m$ starting at initial temperature of 1273K. The results of additional optimisation run to achieve grain size of $30\mu m$ is also presented. Since the extrusion profile (radius and throat length) is a function of velocity of ram and strain, the corresponding trajectories can be used for optimising the extrusion profiles.

The trajectories for the process parameters are obtained for 26 and $30\mu m$. The settling time is found to decrease with increase in grain size and the radius of die at the exit is found to increase with the increase in grain size. The optimal trajectories of strain, strain rate and temperature are shown in **Figure 4**.

9. Process Simulation and Results

Before implementing any algorithm on a real time system, it is advisable to study the performance of the system by simulation. The dynamic model has been developed for 0.3% carbon steel. This model is used for reduction in grain size and volume fraction of recrystallisation. The strain rate in the range of $(0-1.5)s^{-1}$ is given as input to the system and the open loop responses (**Figures 5** and **6**) for grain size and volume fraction of recrystallisation are obtained.

The transfer function of grain size and volume fraction of recrystallisation are obtained as:

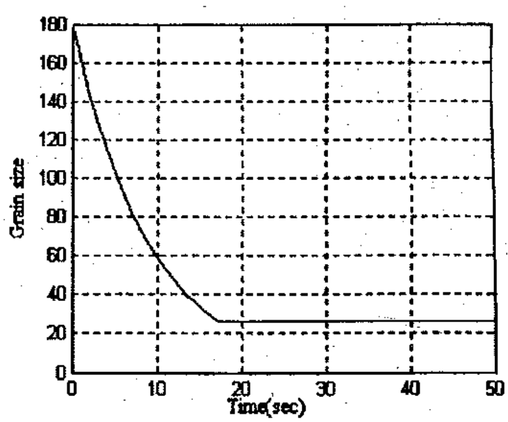

FIGURE 5: *Open Loop Response for Grain Size*

$$G_p(s) = \frac{180}{8.69s+1} \quad \text{........(39)}$$

$$G_p(s) = \frac{e^{-0.425s}}{0.525s+1} \quad \text{........(40)}$$

The PI controller is designed and it is given in **Table 1**.

TABLE 1: *PI Controller Parameters for Grain Size*

K_c	
T_i	

For the same process, GPC parameters are designed and tabulated in **Table 2**.

TABLE 2: *GPC Controller Parameters for Grain Size*

N_1	1
N_2	3
N_u	2

As a next step, PI controller for volume fraction of recrystallisation are obtained and shown in **Table 3**.

TABLE 3: *PI Controller Parameters for Volume Fraction of Recrystallisation*

K_c	1.195
T_i	0.547

GPC parameters for volume fraction of recrystallisation are given in **Table 4**.

TABLE 4: *GPC Controller Parameters for Volume Fraction of Recrystallisation*

N_1	1
N_2	3
N_u	2

By implementing these control algorithms, the closed loop responses (**Figures 7** and **8**) for reduction in grain size of 26 μm and 30 μm are found. Also, the closed responses are determined (**Figures 9** and **10**) for volume fraction of recrystallisation with these controllers.

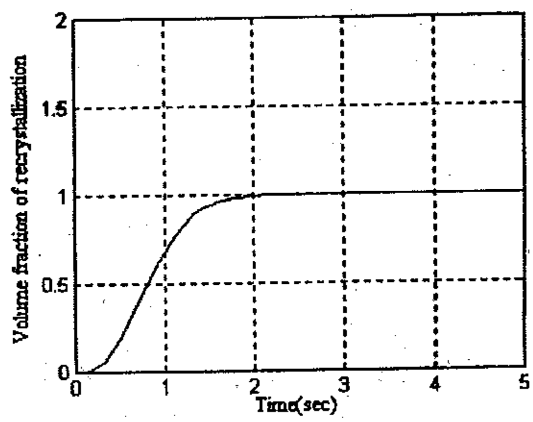

FIGURE 6: *Open Loop Response for Volume Fraction of Recrystallisation*

The optimal ram velocity and die profile for a grain size of 26 μm are shown in **Figures 11** and **12**.

The simple criteria like decay ratio, settling time use only isolated characteristics of the dynamic response. Criterion-like ISE and IAE are based on the entire response of the process. In the present work, performance criteria-like ISE and IAE are calculated for both grain size and volume fraction of recrystallisation. The expressions are given by

$$ISE = \int_0^\infty E^2(t)dt \quad \text{........(41)}$$

$$IAE = \int_0^\infty |E(t)|dt \quad \text{........(42)}$$

The evaluation of performance criteria for a grain size of 26 μm and 30 μm are shown in **Table 5**.

10. Conclusion

The main objective of the control problem presented in this section is to show how easily GPC can be implemented on material-processing and attain better performance. The desired final grain size can be obtained by controlling the strain rate (Input). The state trajectories obtained during the control problem can be used for obtaining the die profile and ram velocity profile. The simulation studies indicate that GPC is superior to widely-accepted, conventional controller and LQR when used on a plant, which has large

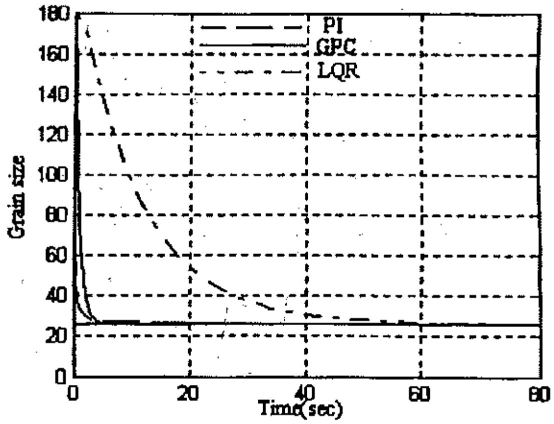

FIGURE 7: *Closed Loop Response for a Grain Size of 26* μm

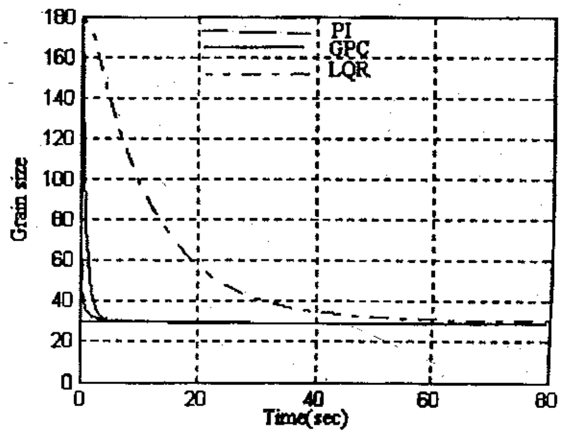

FIGURE 8: *Closed Loop Response for a Grain Size of 30* μm

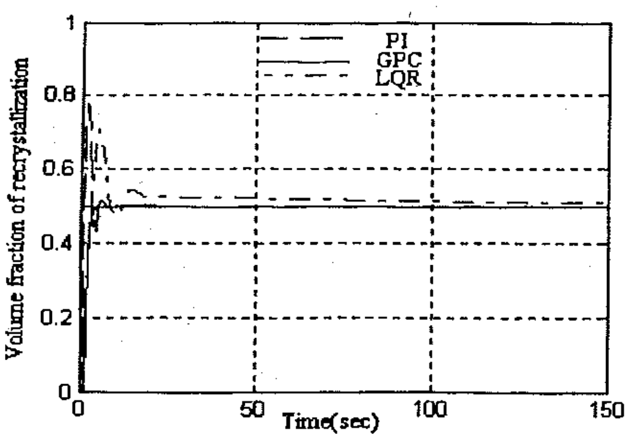

FIGURE 9: *Closed Loop Response for Volume Fraction of Recrystallisation of 0.5*

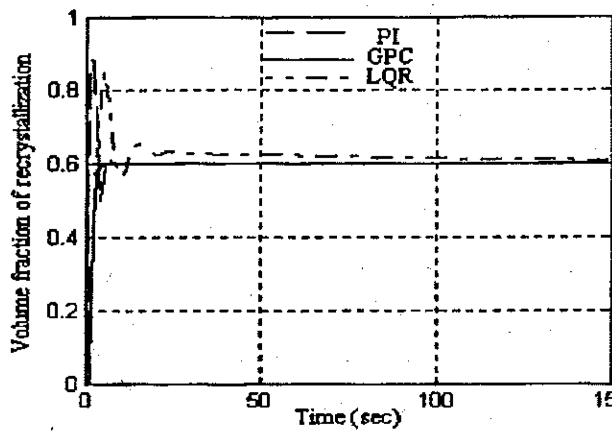

FIGURE 10: *Closed Loop Response for Volume Fraction of Recrystallisation of 0.6*

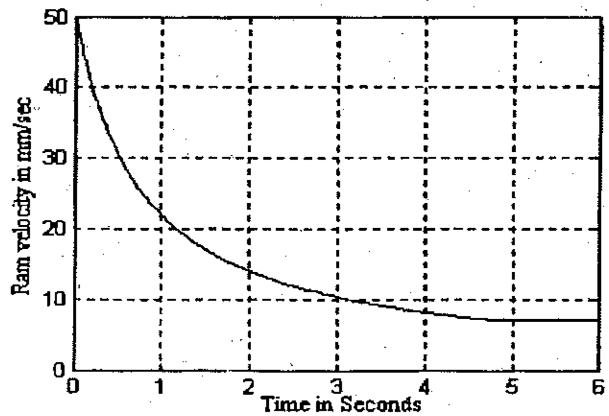

FIGURE 11: *Optimal Ram Velocity for a Grain Size of 26* μm

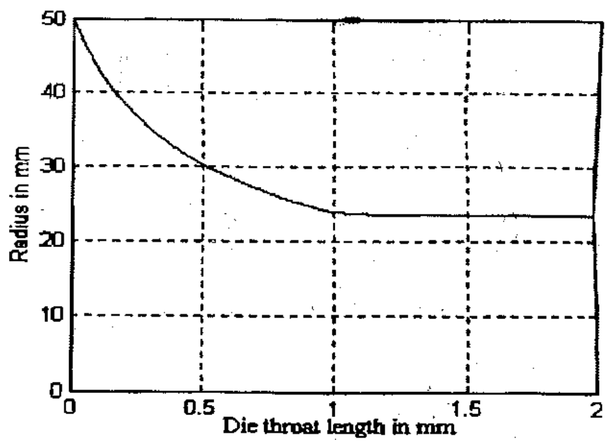

FIGURE 12: *Optimal Die Profile for a Grain Size of 26* μm

TABLE 5: *Performance Criteria for Grain Size*

Set Point	ISE			IAE			Settling Time (sec)		
	PI	GPC	LQR	PI	GPC	LQR	PI	GPC	LQR
26 μm	5.88 X 10^5	2.37 X 10^4	1.59 X 10^5	4.37 X 10^3	3.36 X 10^2	1.99 X 10^3	20	4	70
30 μm	4.52 X 10^5	2.25 X 10^5	1.63 X 10^5	3.88 X 10^3	2.16 X 10^2	1.97 X 10^3	30	4	80

dynamic variations. The GPC algorithm offers satisfactory performance not obtainable with conventional algorithms as evident from **Tables 5** and **6**. Therefore, it can be concluded that the proposed approach using the principles of control theory can be reliably applied to optimise and control the microstructure during extrusion.

11. Summary of the Work

(i) PI, GP Controllers and LQR can be used for microstructural development in material-processing.

(ii) The strain, strain rate, temperature and volume fraction of recrystallisation trajectories can be optimised to achieve the required grain size.

(iii) The strain and ram velocity trajectories can be used to optimise the extrusion profile to achieve the required grain size.

(iv) The simulated results can be validated through experiments.

(v) The desired final grain size can be obtained by controlling the strain rate (input). The state trajectories obtained during the control problem can be used for obtaining the die profile and ram velocity profile.

(vi) The physical and mechanical properties of the materials can be improved by using the proposed methodologies.

List of Symbols

ε - Strain
$\dot{\varepsilon}$ - Strain-rate (sec^{-1})
T - Temperature (K)
ε_c - Critical Strain
$\varepsilon_{0.5}$ - Strain for 50% volume recrystallisation
σ - Flow stress (kPa)
C_p - Specific heat capacity (381 J/Kg K)
ρ - Density (7.8 kg/m^3)
η - Fraction of work, which transforms into heat (0.95)
$\bar{D}$ - Average recrystallised grain size
Q - Activation energy (267 KJ/Mol)
R - Gas constant (8.314 X 10^{-3} KJ /Mol.k)
χ - Volume fraction of recrystallisation

TABLE 6: *Performance Criteria for Volume Fraction of Recrystallisation*

Set Point	ISE			IAE			Settling Time (sec)		
	PI	GPC	LQR	PI	GPC	LQR	PI	GPC	LQR
0.5	3.74	0.517	13.50	8.82	1.16	25.47	20	5	150
0.6	5.39	0.744	13.72	10.69	1.40	26.37	30	5	150

K_c - Controller gain
m_s - Bias
T_i - Integral time constant
$1/T_i$ - Repeats per minute
Kp - Process gain
t_d - Time delay
τ - Time constant
δ - Magnitude of the input change
Δ - Magnitude of the steady-state change in the output

Nomenclature

PI - Proportional and Integral
GPC - Generalised Predictive Control
CARMA - Controlled Auto Regressive and Moving Average
CARIMA - Controlled Auto Regressive and Integral Moving Average
ISE - Integral Square Error
IAE - Integral Absolute Error
LQR - Linear Quadratic Regulator

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