West Indian Journal of Engineering Vol. 28, No. 2, (January 2006) Technical Paper (Baidya & Mandal) 65 - 79

Dynamic Response of Footing Resting on a Layered Soil System

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Dynamic response of foundations depend on several factors such as the shape and size of the foundation, the depth of embedment of the foundation, static and dynamic force levels and the type and extent of soil below the foundation. This paper presents an investigation of the dynamic behaviour of foundations resting on two- and three-layered soil systems. A number of model block vibration tests are carried out on differently-prepared, layered beds using a Lazan-type mechanical oscillator. A steel footing of 0.3m x 0.3m x 0.025m size is used as a model footing. Two types of two-layered beds (a soft layer over a stiff layer and vice versa) and two types of three-layered beds (a soft layer between two stiff layers and a stiff layer between two soft layers) are prepared using sand and sawdust. From the experimental investigations, it is observed that both position and thickness of the layer significantly influence the dynamic response of the foundations. Dynamic response of the foundations on different layered systems is also obtained by elastic, half-space theory representing the layered system by an equivalent homogeneous half-space and then compared with the response obtained from the experimental investigation. Comparisons showed encouraging agreement between the test results and that obtained from the analysis.

Keywords: Footing, resonance, shear modulus, soil-layering, stiffness, vibration.

1. Introduction

The basic goal in the design of foundations with imposed machine-type loading is to limit the amplitude of vibration so that it neither endangers the satisfactory operation of machine nor disturb the people working in the immediate vicinity. This can be achieved by appropriate dynamic analysis of the foundation systems. Over the years, a number of methods have been developed for the analysis of foundation vibration. They are:

- Single degree of freedom mass-spring dashpot model;
- Elastic, half-space theory; and
- Lumped parameter model.

The study of the response of a vertically-loaded, cylindrical disc on an elastic half-space by **Reissner** [1] was probably the beginning in the area of the foundation dynamics. Later, many investigators viz. [2,3,4,5,6,7,8,9,10,11,12,13,14] to name a few, extending **Reissner's** solution studied different modes of vibrations with different contact stress distributions. Although the dynamic response of foundation is greatly influenced by the foundation geometry, rigidity, embedment and soil properties underneath the foundation, most of the analytical solutions are applicable for rigid, circular-footing resting on homogeneous half space.

In reality, however, soils are rarely homogeneous. Soil in a natural state can exist in a state with hard rock at shallow depth, consisting of different strata with different properties (soil-layering). The vertical vibration of a circular footing on the surface of an elastic layer underlain by a rigid base was evaluated by **Bycroft** [4] and **Warburton** [15]. **Warburton** [15] found the effect of an elastic layer on the resonant frequency of the footing for two values of **Poisson's** ratio. The effect of layering on the

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Pertinent discussion will be published in July, 2006 West Indian Journal of Engineering if received by May, 2006.

maximum vibration amplitudes was also evaluated by assuming zero damping in the system. **Gazetas** and **Roesset** [16] have developed a solution for the vertical vibration response of a strip-footing on the surface of an elastic, soil layer, overlying rock. They showed that the presence of a thin layer tends to increase the resonant frequency and amplitude compared to the half-space. A parametric study to examine the effect of soil- layering on vertical vibration response was carried out by **Kagawa** and **Kraft** [17] in which the soil deposit was idealised by a two-layered system; the bottom layer being treated as a half space. They found that:

(1) A large amplitude magnification may be occurring in the layered soil compared to the half-space solution,

(2) When G_2/G_1 exceeds 20, the vibration response is similar to that for an infinitely stiff lower-layer, where G_1 and G_2 is shear modulus of Layer 1 and 2, respectively and

(3) The resonant frequency for a two-layered system is generally less than that for an elastic, half-space.

Sridharan ***et al.*** [14] conducted model block vibration tests on two- and three-layered soil systems and showed the effect of thickness of each layers on resonant amplitude and frequency. Finally, they suggested a method to estimate the equivalent stiffness of the multi-layered soil system using Boussinesq's solution for vertical pressure distribution over depth. **Baidya** and **Sridharan** [18] further improved the method and showed that any multi-layered soil system can be represented by a number of individual springs connected in series from which one can obtain the equivalent stiffness by applying elastic theory. He also showed that the stiffness of any layer was not constant and it was function of position and thickness of the layer.

Many soils exhibit an increase in shear modulus with depth. **Awojobi** [19] developed an approximate solution for the vertical vibration response of a circular footing on the surface of an incompressible soil for which the shear modulus increases linearly with depth with a zero value at the surface of the ground. He found that the footing response on a non-homogeneous, soil system was nearly equal to that of a footing on an elastic, half-space with shear modulus equal to the shear modulus of non-homogeneous soil at a depth equal to the radius of the footing. **Kausel** ***et al.*** [20], **Hadjian** and **Luco** [21], **Ahmad** and **Bhardwaj** [22], **Israil** and **Ahmad** [23], **Borja** and **Wu** [24] and **Wolf** [25] are some of the studies which considered layering in the soil system by various numerical techniques.

Gazetas and **Stoke** [26] discussed different types of experimental investigations related to the vibrating foundations and also discussed the advantage and limitations of each method. They also indicated that case histories and field experiments are the best since the propagation of elastic waves is not interrupted by the presence of artificial, lateral boundaries as in laboratory tests in which spurious wave reflections on the walls may profoundly affect the measured radiation damping. In the literature, however, case histories and field experiments on foundation vibration are limited. A good number of experimental works mostly in the laboratory (viz. **Barkan** [27], **Fry** [28], **Beredugo** and **Novak** [29], **Stoke** and **Richart** [30], **Dobry** ***et al.*** [31], **Nii** [32], **Crouse** ***et al.*** [33], to name a few) on homogeneous soil system are reported in the literature. Experimental work on layered soil system is, however, scanty. Results reported by **Baidya** and **Muralikrishna** [34,35] were probably the first systematic experimental works on layered soil systems. Since in both cases, tests were conducted in a tank, it has some limitations as discussed by **Gazetas** and **Stokoe** [26].

It can be seen from the above review that many investigators studied analytically the dynamic response of the foundations resting on both homogeneous half-space and non-homogeneous systems. However, literature describing the actual behaviour of vibrating foundations based on experimental investigations is scanty and there is ample scope for both analytical and experimental investigations to improve the understanding of dynamic response of foundations on non-homogeneous (layered) soil system. Hence, in this paper an attempt is made to study the effect of soil-layering on the dynamic response of the foundations by model block vibration tests conducted in a pit. Four types of soil-layering are considered in the investigation. They are:

(1) A soft layer over a stiff layer;

(2) A stiff layer over a soft layer;

(3) A soft layer between two stiff layers; and

(4) A stiff layer between two soft layers.

Furthermore, this study includes:

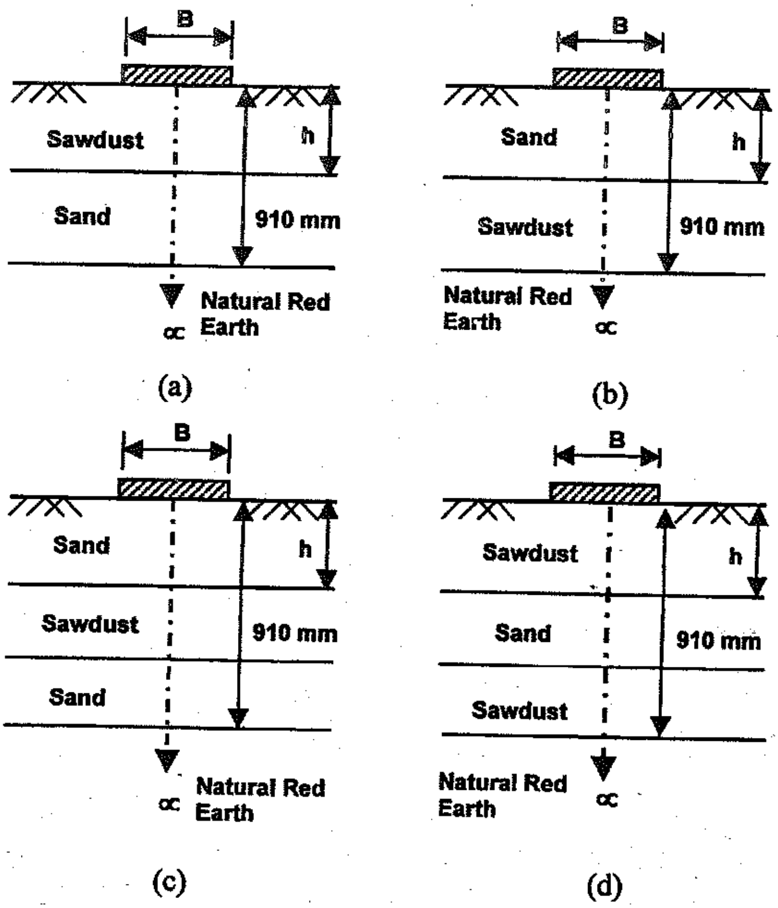
(a) Investigation of the effect of thickness and position of layers on resonant frequency and amplitude;

(b) Dynamic analysis of the foundations resting on various layered soil system by equivalent half space theory; and

(c) Verification of the adequacy of the equivalent half space theory for layered soil system comparing with the experimental results.

2.0 Experimental Programme

The effect of layering on the dynamic response of foundation is investigated experimentally by conducting vertical vibration tests using a Lazan- type, mechanical oscillator. The mechanical oscillator consists of two shafts (containing eccentric mass) so arranged that they rotate in opposite directions at the same speed when one of them is driven by a motor through a flexible shaft. Such an arrangement induces a vibratory force at the base in the vertical direction. Three different static weights, W (3.5, 4.1 and 4.7kN) were used to simulate three different static masses and three different eccentric settings, $m_e e$ (0.1,



(a) (b) (c) (d)

FIGURE 1: *Experimental Programme: (a) Soft Layer over a Stiff Layer and (b) Stiff Layer over a Soft Layer (c) Soft Layer between Two Stiff Layers and (d) Stiff Layer between Two Soft Layers*

0.2 and 0.3 N-sec^2) were used to simulate three different dynamic force magnitudes. The detailed programme for the study is presented in **Figure 1**.

3.0 Test Set-up and Test Procedure

3.1 Test Pit

In the investigation, it is proposed to study the effects of variation of the top and bottom layer thickness and of the position of the layer on the dynamic response. This requires a study on a large numbers of layering combinations as shown in **Figure 1(a)–1(d)**. The proposed layered beds were prepared in a pit of size 1.2 m x 1.2 m x 0.91 m dug in the field which consists of a thick layer of red earth. Since the 1970s, a large number of footing vibration tests have been carried out in pits and tanks to study various aspects of machine foundation systems at the Indian Institute of Science, Bangalore, India,. Suitability of the dimensions of the pits and tanks with respect to the size of the footing for possible boundary effects were checked each time. **Raman** [36] conducted footing vibration tests with a footing of size 0.23 m-diameter in the pits of size 0.76 m x 0.76 m x 0.76 m and 1.06 m x 1.06 m x 0.91 m. Responses obtained from both the pits were compared and found almost identical. Based on this experience, the entire tests in this study were conducted in a pit of size 1.2m x 1.2m x 0.91m using a steel footing of size 0.3m x 0.3m x 0.025m.

3.2 Test Materials

Sawdust and sand were chosen as test materials to form the layered beds. These were chosen with an aim to investigate the effect over a wide range of soil (sand as moderately stiff to sawdust as extremely soft). Further, these materials were chosen because of the fact that they were easy to work with and maintain uniformity while preparing the layered beds.

3.3 Preparation of Layered Beds

Medium fine river sand (ϕ = 36° from direct shear test, $\gamma \cong 17$ kN/m^3) and locally-available sawdust ($\gamma = 2.3$ kN/m^3) were used to form the proposed layered system. To maintain uniformity throughout the test programme, the pit was filled in steps of 0.15 m-thick layer and each layer was compacted using a constant compactive effort to achieve a density approximately 17 kN/m^3 for sand. Sawdust is a very difficult material and dropping weight does not work for compacting it. It was compacted by walking over it in several passes and applying static weight for some time. The density of sawdust achieved by this way was 2.3 kN/m3 and this density was fairly maintained in the entire investigation. For the first set of beds used in the study, the 0.91 m-thick sand bed was prepared in six steps of 0.15 m each. After completing the desired test on this bed, sand from the top (0.15 m-thick) was replaced by sawdust. By this process, the thickness of the sawdust layer at the top was increased approximately in steps of 0.15 m up to 0.91 m. Similarly, starting with a sawdust bed of thickness 0.91-m, the thickness of the sawdust layer was then reduced and the thickness of the sand layer was increased by replacing sawdust from top by sand in steps of 0.15-m up to 0.91m. It was planned to vary the thickness of the layer by approximately half the width of the footing. However, the thicknesses achieved during investigation were slightly different. Thicknesses of the layers achieved in the two-layered system during investigation are presented in **Table 1**. Two different types of the three-layered system were also prepared using sand and sawdust at different positions as shown in **Table 2**.

3.4 Test Procedure

The steel footing was first placed centrally over a prepared layered bed. A mechanical oscillator was then placed over the plate and steel ingots were placed on the top of the oscillator to provide the required static weight. Sufficient rubber packing between the two ingots was placed for tight fixing. The whole set-up was then rigidly connected to act as a unit mass during vibration. Proper care was taken to maintain the centre of gravity of the whole system and the footing in the same vertical line. Initially, a static weight, W = 3.5 kN, and eccentric setting, $m_e e$ =0.1 N-sec^2 were set. The oscillator was then connected to a variable speed DC motor (3 H.P. range up to 50 Hz) through a flexible shaft. A B&K piezoelectric-type, accelerometer pickup was placed on top of the footing to measure the amplitude using a B&K vibration meter. The oscillator was then run slowly through a motor using a speed control unit. The foundation was thus subjected to vibration in the vertical mode. Frequencies and corresponding displacement amplitudes of vibration were recorded by a phototachometer and the vibration meter

TABLE 1: *Detailed Experimental Programme for Two-layered System*

Layer 1		Layer 2		Layer 3
Material	Thickness (mm)	Material	Thickness (mm)	Material/Thickness
Sawdust	0.00	Sand	910	Natural red earth beyond 910 mm depth
	165		745	
	310		600	
	460		450	
	610		300	
	910		000	
Sand	000	Sawdust	910	Natural red earth beyond 910 mm depth
	160		750	
	335		575	
	490		420	
	645		265	
	910		000	

respectively. To obtain a complete foundation response (frequency vs. displacement amplitude curve) and to locate the resonant peak accurately, the amplitudes were noted at a frequency interval of 30–60 rpm (0.5–1.0 Hz). Subsequently, eccentric setting was increased to 0.2 and 0.3 N-sec^2 for the same static weight. The same tests were repeated with static weights of 4.1 kN and 4.7 kN. **Figure 2** shows the schematic diagram of complete experimental set-up.

4.0 Experimental Results

In each layered bed, a total of nine tests were conducted (three static weights and for each static weight three eccentricities). Hence, a total of nine frequency vs. displacement amplitude curves were obtained for each bed and presented in a single figure. **Figure 3** presents frequency-amplitude curves from a two-layered system for which thickness of sawdust layer at top is 610 mm (thickness of sand layer at bottom is 300 mm). It can be seen from **Figure 3** that with the increase in eccentric setting, $m_e e$, the resonant frequency decreases and resonant amplitude increases for constant value of static weight. For example, resonant frequency decreased from 655 rpm (10.9 Hz) to 625 rpm (10.4 Hz) and resonant amplitude increased from 200 microns to 490 microns for increase of eccentric

TABLE 2: *Detailed Experimental Programme for Three-layered System*

Layer 1		Layer 2		Layer 3		Layer 4
Material	Thickness (mm)	Material	Thickness (mm)	Material	Thickness (mm)	Material/Thickness
Sand	0	Sawdust	610	Sand	300	Natural red earth
	150		460		300	beyond 910 mm
	340		270		300	depth
	470		140		300	
Sawdust	0	Sand	645	Sawdust	265	Natural red earth
	140		505		265	beyond 910 mm
	310		335		265	depth
	645		0		265	

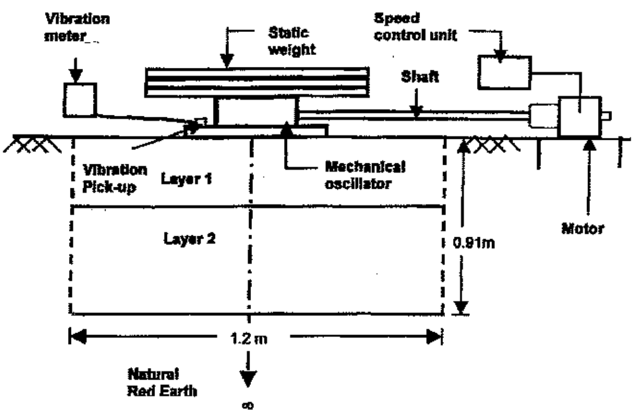


FIGURE 2: *Schematic Diagram of Experimental Setpup*

setting from 0.1 to 0.3 N-sec^2 for static weight of 4.1 kN. Further, it can be seen from **Figure 3** that with the increase in static weight, both resonant frequency and amplitude decreases. For example, resonant frequency decreased from 655 rpm (10.9 Hz) to 625 rpm (10.4 Hz) and resonant amplitude from 440 microns to 300 microns due to increase of static weight from 3.5 kN to 4.7 kN for $m_e e = 0.2$ N-sec^2. These observations qualitatively agree with that reported by past investigators (**Fry** [28], **Moore** [37], **Richart *et al.*** [8]). Hence, results corresponding to static weight of 4.1 kN and eccentric setting of 0.2 N-sec2 are presented in this paper to investigate the effect of layering on the dynamic response. **Figure 4** presents the variation of response with the variation of thickness of the sawdust layer at the top keeping total thickness constant. It can be seen from **Figure 4** that the resonant frequency decreased from 1825 rpm (30.4 Hz) to 890 rpm (14.8 Hz) when 165 mm-thick sand from the top is replaced by sawdust. However, due to further increase of thickness of sawdust layer at the top, the reduction in resonant frequency is not that significant and it is almost constant when the thickness of sawdust layer at top is more than 610 mm. The displacement amplitude of vibrations however, consistently increased due to the increase of thickness of sawdust layer at the top. The decrease of resonant frequency and increase of resonant amplitude is the result of decreasing stiffness and damping with the increase of thickness of sawdust at top. Since, the variation of response at a depth of 910 mm compared to the previous depth (610 mm) is practically zero, the response on a 910 mm-thick sawdust layer is

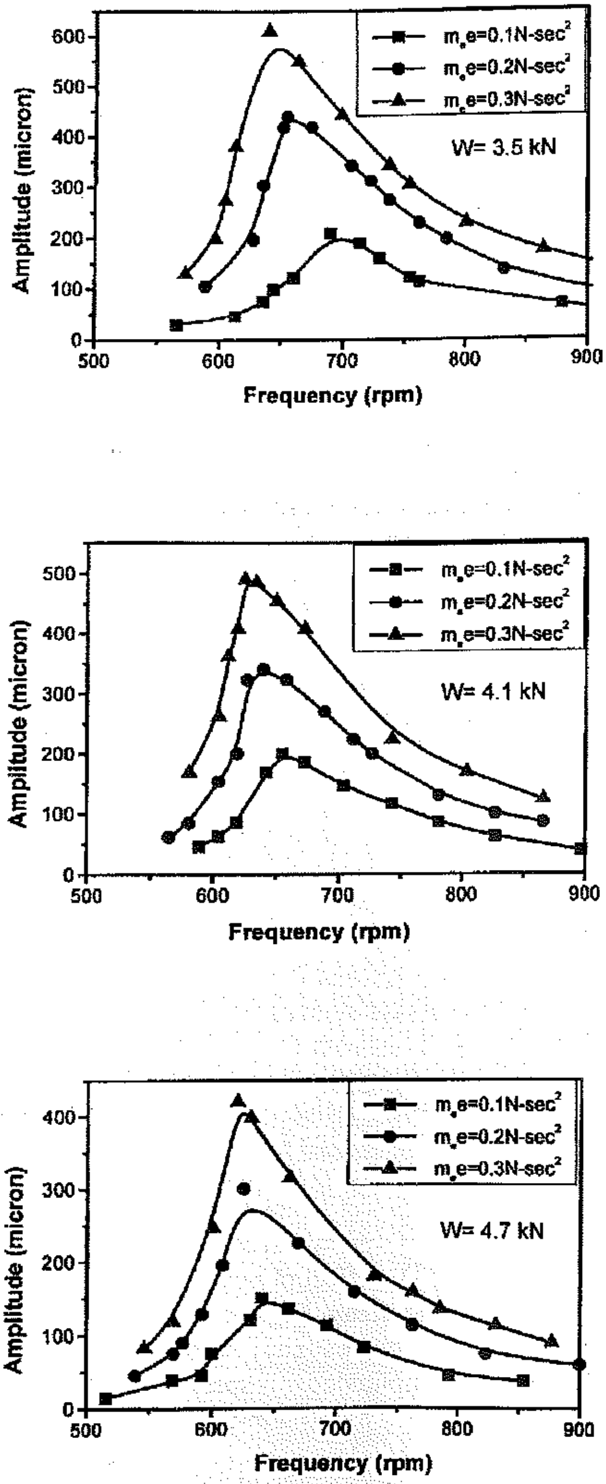


FIGURE 3: *Frequency vs. Displacement Amplitude Curves from a Two-layered Soil System (Thickness of Sawdust Layer at Top is 460 mm); (a) For Static Weight 3.5 kN, (b) For Static Weight 4.1 kN and (c) For Static Weight of 4.7 kN*

considered as the response on a half space. Hence, equating theoretical natural frequency of half space, $\sqrt{K_h / m}$, with the resonant frequency, f_r, obtained from the test on a 920 mm-thick sawdust layer, shear modulus of the sawdust is obtained as

$$G = \frac{(1-\nu)\, f_r^2 m}{4r} \qquad (1)$$

Where K_h is the stiffness of homogeneous elastic, half-space $= \frac{4Gr}{(1-\nu)}$, m is the total vibrating mass, r is the radius of the circular footing, or radius of the equivalent circle for non-circular footing and ν is the Poisson's ratio. Assuming Poisson's ratio of sawdust as 0.0, average value of shear modulus of sawdust is obtained as 1.75 MN/m^2.

Figure 5 presents the variation of response with the variation of thickness of sand layer at top. It can be seen from **Figure 5** that with the increase of thickness of sand layer at top resonant frequency gradual increases [555 rpm (9.3 Hz) to 1825 rpm (30.4 Hz) for sand layer thickness variation from 0.0 to 910 mm] and peak displacement amplitude gradually decreases (430 microns to 155 microns). This is due to increase of stiffness and damping with the increase of sand layer thickness at top. Red earth at the site is qualitatively similar (in terms of density and shear wave velocity) to that of sand. Hence, when the pit is filled up with sand (910 mm-thick), it is treated as half-space and shear modulus of sand is obtained similarly as it is done for sawdust (Equation 1). Assuming the Poisson's ratio of sand as 0.3, the average value of shear modulus of sand obtained from Equation 1 was 13.85 MN/m^2. The effect of position of the layer can be found comparing **Figure 4** with **Figure 5**. For example, the resonant frequency and peak displacement amplitude are respectively, 640 rpm and 335 microns when a 460 mm-thick sawdust layer at the top and it is 1340 rpm and 125 microns when the sawdust layer of almost same thickness is at 490 mm-depth.

Figure 6 presents results of the three-layered soil system (a soft layer between two stiff layers). In this, thickness of the bottommost layer was kept constant (300 mm) and thickness of the top two layers were varied keeping total thickness constant (910 mm).

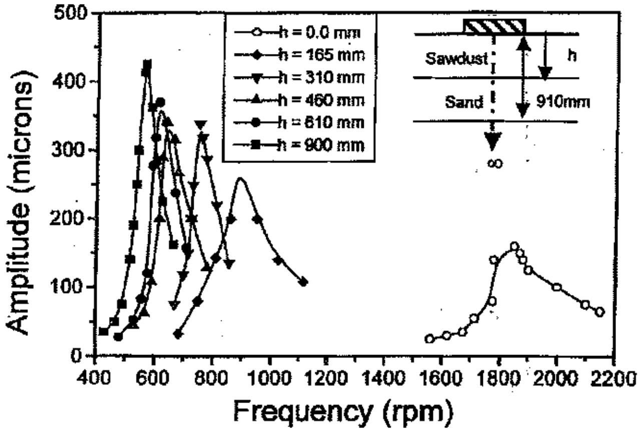


FIGURE 4: *Variation of Response with the Variation of Thickness of Sawdust Layer at Top [Figure 1 (a)]: Static Weight = 4.1 kN and Eccentric Setting = 0.2 N-sec^2*

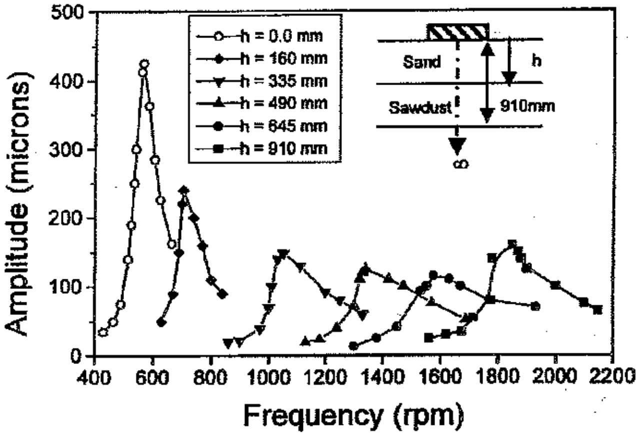


FIGURE 5: *Variation of Response with the Variation of Thickness of Sand Layer at Top [Figure 1 (b)]: Static Weight = 4.1 kN and Eccentric Setting = 0.2 N-sec^2*

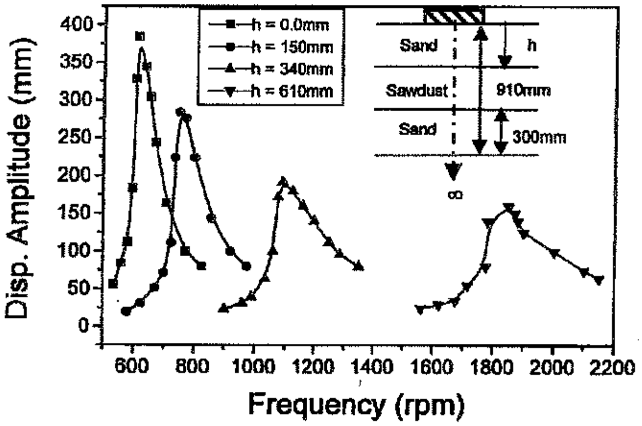


FIGURE 6: *Variation of Response with the Variation of Thickness of Sand Layer at Top [Figure 1 (c)]: Static Weight = 4.1 kN and Eccentric Setting = 0.2 N-sec^2*

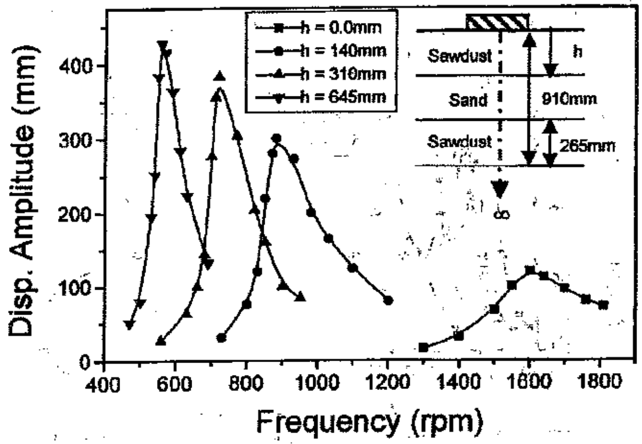


FIGURE 7: *Variation of Response with the Variation of Thickness of Sawdust Layer at Top [Figure 1 (d)]: Static Weight = 4.1 kN and Eccentric Setting = 0.2 N-sec²*

It can be seen from **Figure 6** that with the increase of sand layer at the top, resonant frequency increases and peak displacement amplitude decreases. With the increase of thickness of sand layer at top, the soil system becomes stiffer resulting in increase of resonant frequency and decrease of peak displacement. **Figure 7** presents similar results for a three-layered system where a stiff layer is between two soft layers. It can be seen from **Figure 7** that with the increase in thickness of sawdust layer at top, resonant frequency decreases and peak displacement increases. With the increase of thickness of soft layer at top, the soil system becomes less stiff and results in decrease of resonant frequency and increase of peak displacement. The effect of positions of layers can also be seen comparing **Figure 6** with **Figure 7**. From these observations, it can be concluded that the position and thickness of the layer have a significant effect on the dynamic response of the foundation. An attempt is made to provide a simple, approximate solution incorporating the effect of position and thickness of the soil layer in the dynamic analysis of the foundations resting on different layered soil systems in the following section.

5.0 Analysis of Foundation Response

5.1 Analytical Consideration

Elastic half space theory developed by **Bycroft** [4] has been effectively used for the analysis of vibrating footing. Many investigators have extended and simplified his work for practical use. An expression for the magnification factor for vertical vibration was given by (**Lysmer** and **Richart** [6]):

$$M_r = B_z a_o^2 \left[\frac{F_1^2 + F_2^2}{\left(1 - B_z a_o^2 F_1\right)^2 + \left(B_z a_o^2 F_2\right)^2} \right]^{1/2} \tag{2}$$

Where a_o = Dimensionless frequency ratio $= \varpi r_o \sqrt{\rho / G}$, B_z = Modified mass ratio $= \frac{(1-\nu)}{4}\frac{m}{\rho r^3}$, F_1 and F_2 are displacement functions.

Fitted equations for displacement functions are given below (**Sridharan *et al.*** [38]):

$$F_1 = 1.0 - 0.517361 a_o^2 + 0.108376 a_o^4$$

$$F_2 = 0.517638 a_o - 0.260983 a_o^3 + 0.038301 a_o^5$$

Using Equation 2, resonant frequency and peak displacement of the foundation soil system resting on a half-space can be obtained. In the present investigation, the applicability of this method for dynamic response analysis of the foundations resting on layered soil systems is verified. For this purpose, the layered system is transformed into an equivalent homogeneous system.

Different methods (arithmetic average and weighted average) have been used for obtaining the equivalent properties of layered soil systems. In arithmetic average, neither the position nor the thickness of the soil layer is considered. In weighted average method, the effect of thickness is considered, whereas, the importance of position is still neglected. It is found from the experimental investigations that both the position and the thickness of the soil layer have significant influence on the dynamic response of the foundation resting on layered soil system. Hence, a method is adopted to estimate the equivalent properties of layered soil systems considering their position and thickness.

5.2 Equivalent Shear Modulus

Prediction of natural frequency of vibrating foundations resting on a finite stratum and on a layered soil based on static equivalent stiffness was found satisfactory (**Baidya** and **Muralikrishna** [34,35]). Equivalent stiffness of the rigid footing resting on a multi-layered soil system was given by **Baidya** and **Muralikrishna** [35] as below

$$K_e = \frac{1}{\sum_{i=1}^{n} 1/K_i} \tag{3}$$

Where K_i is the stiffness of the i-th layer $= \dfrac{\pi G_i r_o}{\left[F\right]_{z_{i-1}/r_o}^{z_i/r_o}}$,

G_i is the shear modulus of the i-th layer, F is the depth function as given below (**Baidya *et al.*** [39]):

$$F = \left[\frac{(1-\nu)}{2}\tan^{-1}(z/r_o) - \frac{1}{4}\frac{(z/r_o)}{\left(1+z^2/r_o^2\right)}\right]$$

and r_o is the radius of footing or radius of equivalent circle for non-circular footing.

Equating equivalent stiffness of the multi-layered soil system from Equation 3 with stiffness of homogeneous half space from elastic theory [i.e., $K = 4Gr_o/(1-\nu)$], the shear modulus of the equivalent half space can be obtained. Prior to this calculation, Poisson's ratio for the equivalent system has to be estimated. Poisson's ratio affects both the resonant frequency and the amplitude of vibration (**Sulfi** [40]). The amplitude of vibration decreases as Poisson's ratio increases but the resonant frequency increases as Poisson's ratio increases. The range of ν is, however, very narrow (0.25-0.45) for a wide range of soils. Hence, for practical purpose, ν can be used as 0.3-0.4 depending on type of soil. However, equivalent Poisson ratio, ν_e for the layered soil system is estimated in the present study from the equation given below:

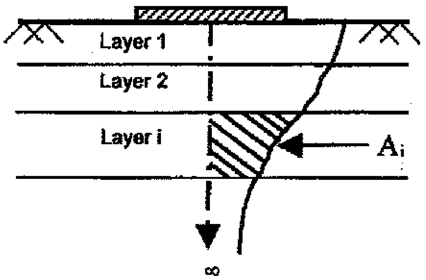


FIGURE 8: *Boussinesq's Influence Diagram for the Calculation of Equivalent Soil Properties*

$$\nu_e = \frac{\sum_{i=1}^{n} A_i \nu_i}{\sum_{i=1}^{n} A_i} \tag{4}$$

Where $A_i = r_o F_z$, is the area of the influence diagram under layer i as shown in **Figure 8**,

$$F_z = \left[2\tan^{-1}(z/r_o) - \frac{z/r_o}{1+z^2/r_o^2}\right]$$

(**Baidya** [41]) and ν_i is the Poisson's ratio of i-th layer.

Similarly, equivalent unit weight, γ_e, of the layered soil system is obtained as,

$$\gamma_e = \frac{\sum_{i=1}^{n} \gamma_i A_i}{\sum_{i=1}^{n} A_i} \tag{5}$$

Where γ_i is the unit weight of the i-th layer.

Obtaining equivalent properties of the multi-layered soil system from Equation 3 through 5 and then using it in Equation 2, the complete response of the foundations can be obtained. Using shear

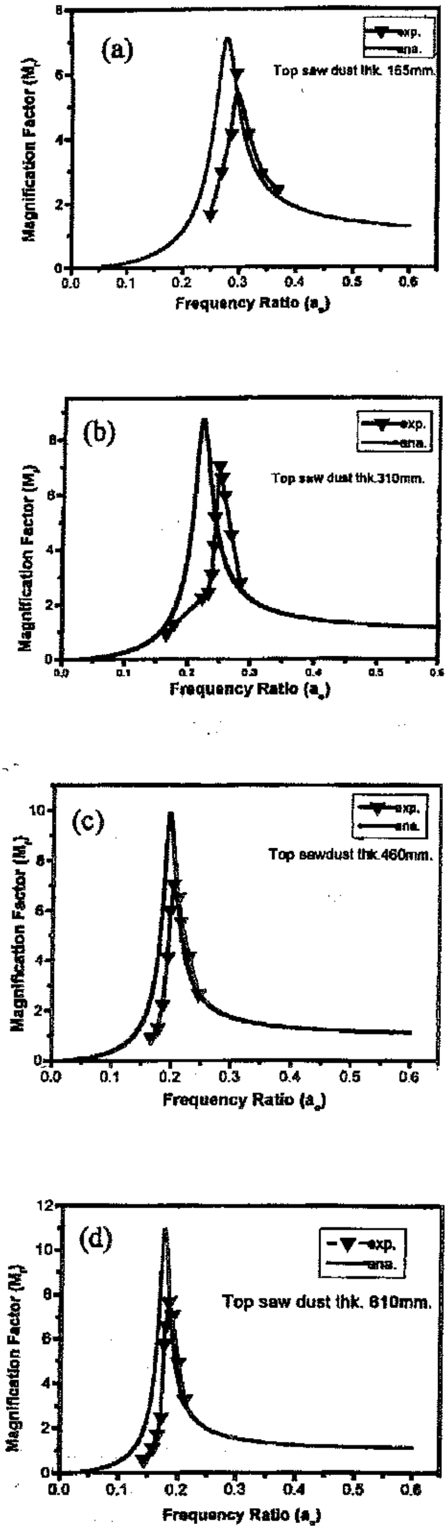


FIGURE 9: *Comparison between the Response obtained from the Equivalent Half-space Method and that obtained from the Experimental Investigation for Two-layered System; Sawdust Layer Thickness at Top (a) 165 mm, (b) 310 mm, (c) 460 mm and (d) 610 mm*

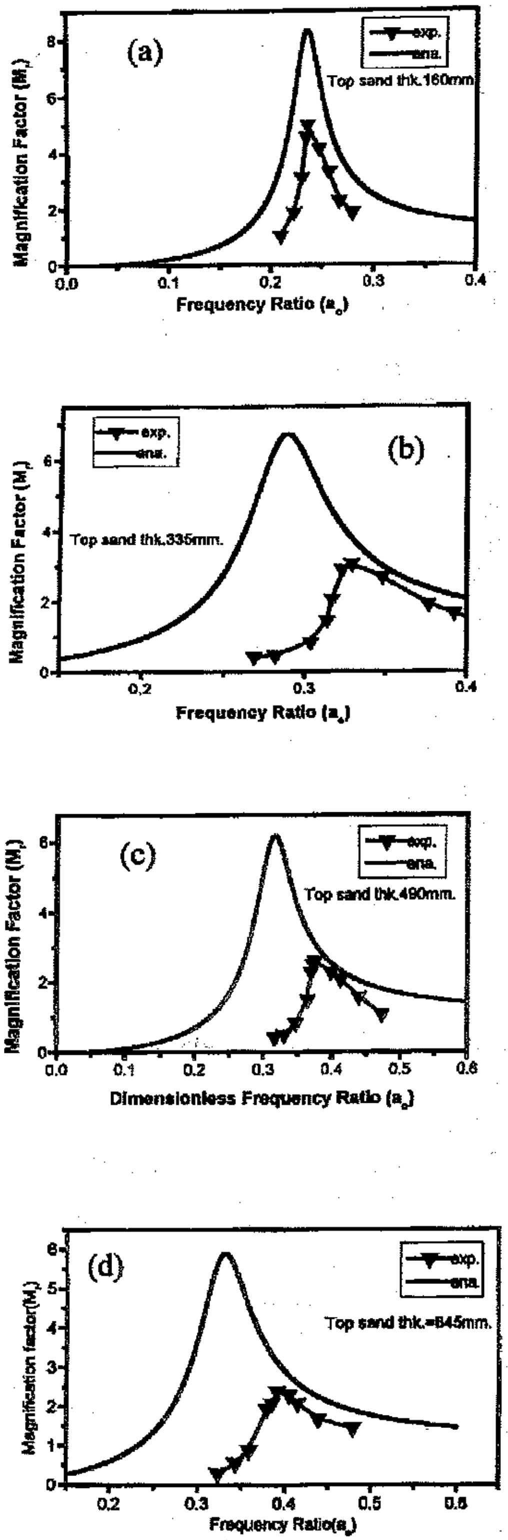


FIGURE 10: *Comparison between the Response obtained from the Equivalent Half-space Method and that obtained from the Experimental Investigation for Two-layered System; Sand Layer Thickness at Top (a) 160 mm, (b) 335 mm, (c) 490 mm and (d) 645 mm*

modulus values of sand and sawdust obtained from experimental result previously, equivalent properties of different layered soil systems as presented in **Tables 1** and **2** are obtained. Finally, dynamic responses of the equivalent systems corresponding to different layered soil systems are obtained using Equation 2.

6. Comparisons

Dynamic response of the foundation resting on the layered soil systems is obtained in the non-dimensional form representing the multi-layered soil system into an equivalent system. Experimental results are also expressed in non-dimensional form for comparison. **Figures 9 (a)** through **9 (d)** present the comparisons for the two-layered systems when the top layer is softer than the bottom whereas **Figures 10 (a)** through **10 (d)** present comparisons for the two-layered systems when the top layer is stiffer than the bottom layer. **Figure 11 (a)** through **11 (d)** present comparisons for the three-layered soil system with the middle layer softer than the other two layers whereas **Figure 12 (a)** through **12 (d)** present comparisons for the three-layered soil system with the middle layer stiffer than the other two layers. It can be seen from **Figures 9** through **12** that the experimental results (resonant frequency) compare well with the analytical results except for the cases when a thick, stiff layer is at the top. Thick sand layer at top adds extra surcharge (γh, since γ_{sand} is approximately 8 times the $\gamma_{sawdust}$) and confinement to the sawdust layer below it which results in an increase in shear modulus of the sawdust. However, this effect could not be considered while estimating the shear modulus of the sawdust and hence, the differences. Range of unit weight for a wide range of soil is approximately between 15 kN/m^3 and 22 kN/m^3. However, when two natural soil forms a layered system, this effect will be insignificant (ratio of densest and loosest soil density is around 1.5). Hence, it can be concluded from the above discussions that the proposed method may give better results for a natural layered system than found in the experimental investigation.

Resonant amplitudes from experiments are found to be consistently lower compared to that obtained from the analysis. Actual damping of the vibrating foundation systems consists of two parts, namely radiation damping and material damping. Material damping for the wide range of soils varies

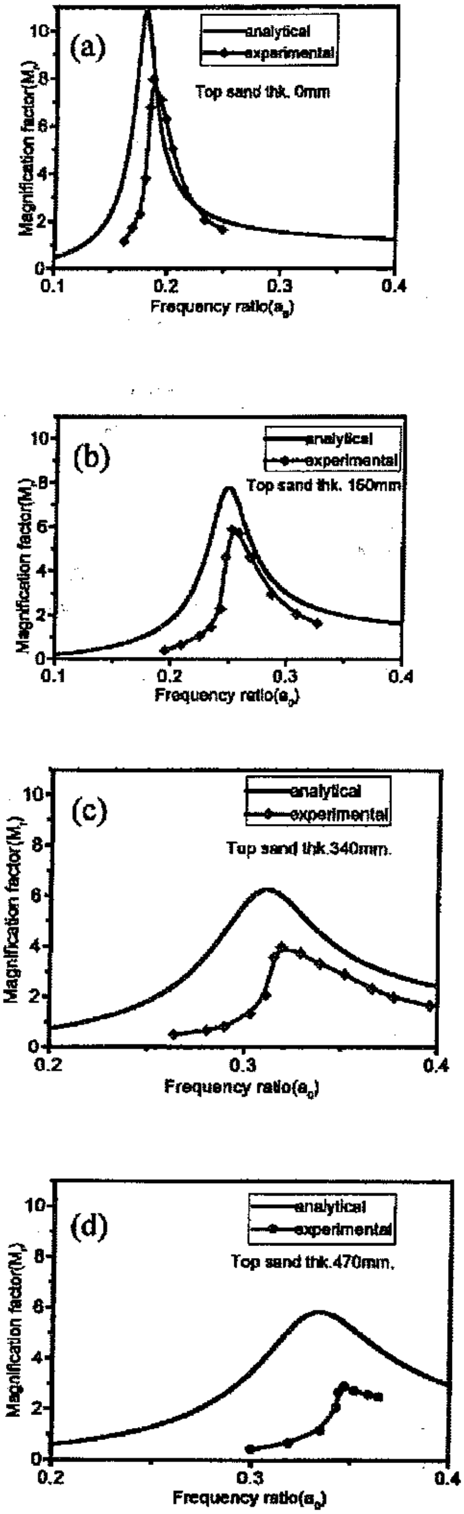


FIGURE 11: *Comparison between the Response obtained from the Equivalent Half-space Method and that obtained from the Experimental Investigation for Three-layered System; Sand Layer Thickness at Top (a) 0.0 mm, (b) 150 mm, (c) 340 mm and (d) 470 mm*

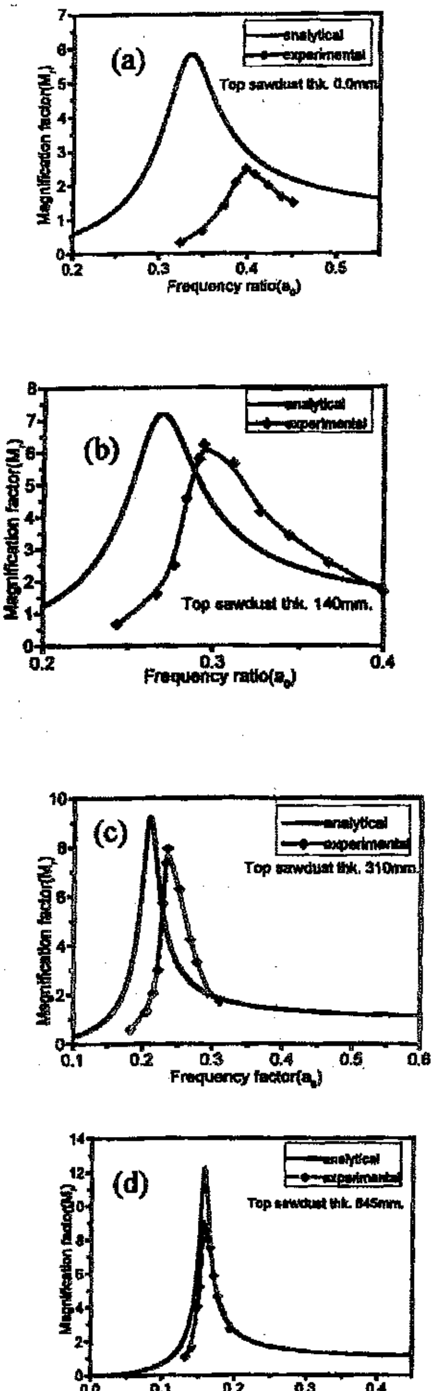


FIGURE 12: *Comparison between the Response obtained from the Equivalent Half-space Method and that obtained from the Experimental Investigation for Three-layered System; Saw Dust Layer Thickness at Top (a) 0.0 mm, (b) 140 mm, (c) 310 mm and (d) 645 mm*

between 3 and 8% of the critical damping and can be estimated by laboratory test. Radiation damping can be as high as 50% and can be estimated approximately as $0.425/\sqrt{B_z}$ for the vertical mode of vibration (**Richart** ***et al.*** [8]). Elastic half-space theory considers only radiation damping which is less than the actual damping. Hence, the resonant amplitudes predicted are consistently higher (underestimating damping) compared to that obtained from the experiments.

7. Conclusions

Model block, vibration test results of footings resting on different, layered soil systems are presented. It is observed from the experimental results that with the increase in thickness of soft layer at top, resonant frequency decreases and resonant amplitude increases whereas with increase in thickness of stiff layer at top resonant frequency increases and resonant amplitude decreases. Further, it is observed that the dynamic response is significantly different when a layer of finite thickness is at top than when it is at a depth. Hence, dynamic response of the foundations resting on a layered, soil system depends both on the position and thickness of the layer.

All the layered systems studied in the experimental investigations are analysed using elastic half-space theory with equivalent soil properties for the layered, soil system and compared with the experimental results. In total, 16 layered systems [(i) a soft layer over a stiff layer – 4 nos; (ii) a stiff layer over a soft layer – 4 nos., (iii) a soft layer between two stiff layers – 4 nos. and (iv) a stiff layer between two soft layers – 4 nos.] are compared with analytical results. Comparison showed a good agreement between the experimental results and that obtained from the equivalent half-space theory. Hence, an overall conclusion can be drawn from the present study that the elastic half-space theory with the equivalent half-space properties is satisfactory for the dynamic analysis of the foundation resting on the layered, soil systems. Results presented in this paper may also prove particularly useful for the preliminary calculation in the conceptual stages of design process.

List of Notations

A	Area of influence diagram
a_o	Dimensionless frequency raio = $\varpi r_o \sqrt{\rho / G}$
B_z	Modified mass ratio = $\frac{(1-\nu)}{4} \frac{m}{\rho r_o^3}$
E	Eccentricity of the rotating mass
F_1 & F_2	Displacement functions
G_e	Equivalent shear modulus of the layered soil
G_i	Shear modulus of i-th layer
K	Stiffness of foundation resting on a half-space
K_i	Stiffness of the i-th layer
K_e	Equivalent stiffness
M_r	Magnification factor
m	Total vibrating mass
$m_e e$	Eccentric setting
r_o	Radius of footing or radius of equivalent circle for non-circular footing
γ_e	Equivalent unit weight of the soil
ν_e	Equivalent Poisson's ratio of the layered soil
ϖ	Circular frequency of vibration

References

[1] Reissner, E. (1936). *Stationare, Axial-symmetrische, Durch eine Schut-telnde Masse Erregte Schwingungen eines Homogenen Elastischen Halbraumes.* Ingenieur Archiv, Berlin, Germany 7(6): pp. 381–396.

[2] Sung, T.Y. (1953). *Vibrations in Semi-infinite Solids due to Periodic Surface Loadings.* Symp. on Dynamic Testing of Soils, STP No. 156, ASTM, Philadelphia, pp. 35–64.

[3] Quinlan, P.M. (1953). *The Elastic Theory of Soil Dynamics.* Symposium on Dynamic Testing of Soil, ASTM, STP No. 156: pp. 3–34.

[4] Bycroft, G.N. (1956). *Forced Vibrations of a Rigid Circular Plate on a Semi-infinite Elastic Half-space and on an Elastic Stratum.* Philosophical Transactions of Royal Society of London, 248(948): pp. 327–368.

[5] Richart, F.E. Jr. (1960). *Foundation Vibrations.* Journal of Soil Mechanics and Foundation Engineering, ASCE, 86(SM 4): pp. 1–34.

[6] Lysmer, J. and Richart, F.E., Jr., (1966). *Dynamic Response of Footings to Vertical Loading.* Journal Soil Mechanics and Foundation Engineering Division, ASCE, 92 (SMI): pp. 65–91.

[7] Richart, F.E. and Whitman, R.V. (1967). *Comparison of Footing Vibration Test with Theory.* Journal of Soil Mechanics and Foundation Engineering, ASCE, 93(SM 6): pp. 143–168.

[8] Richart, F.E. Jr., Hall, J.R. Jr. and Woods, R.D. (1970). *Vibrations of Soils and Foundations.* Printice-Hall, Inc. Englewood Cliffs, New Jersey.

[9] Luco, J.E. and Westman, R.A. (1971). *Dynamic Response of Circular Footings.* Journal Engineering Mechanics, ASCE, 97 (EM-5): pp. 1381–1395.

[10] Sridharan, A. and Raman, J. (1976). *Prediction of Dynamic Response of Footing Soil System.* Proc. 5th Conf. Soil Mechanics and Foundation Engineering, pp. 423–434, Budapest.

[11] Nagendra, M.V. and Sridharan, A. (1982). *Stiffness Coefficient of Elastic Medium.* Proceedings of Journal of Geotechnical Engineering, ASCE, 108(GT4): pp. 661–668.

[12] Nagendra, M.V. and Sridharan, A. (1984). *Footing Response to Horizontal Vibration.* Journal of Engineering Mechanics, ASCE, 110(4): pp. 648–654.

[13] Nagendra, M.V., Sridharan, A. and Sreenivasan, M. (1984). *Fondation Response to Horizontal Vibrations.* Indian Geotechnical Journal, 12: pp. 132–151.

[14] Sridharan, A., Gandhi, N.S.V.V.S.J. and Suresh, S. (1990). *Stiffness Coefficients of Layered Soil System.* Journal of Geotechnical Engineering, ASCE, 116(4): pp. 604–624.

[15] Warburton, G.B. (1957). *Forced Vibration of a Body on an Elastic Stratum.* Journal Applied Mechanics, Transaction ASME. 24, pp. 55–58.

[16] Gazetas, G. and Roesset, J.M. (1979). *Vertical Vibration of Machine Foundations.* Journal of Geotechnical Engineering, ASCE, 105(12): pp. 1435–1454.

[17] Kagawa, T. and Kraft, L.M. (1981). *Machine Foundations on Layered Soil Deposits.* Proceedings 10th Int. Conf. Soil Mechanics and Foundations Engineering, Stockholm, 3: pp. 249–252.

[18] Baidya, D. K. and Sridharan, A. (2002). *Foundation Vibration on Layered Soil System.* Indian Geotechnical Journal, 32 (3): pp. 235–257.

[19] Awojobi, A. 0. (1972). *Vertical Vibration of a Circular Foundation on Gibson Soil.* Geotechnique, 22(2): pp. 333–343.

[20] Kausel, E., Rosset, J.M. and Wass, G., (1975). *Dynamic Analysis of Footing on Layered Media.* Journal of Engineering Mechanics, ASCE, 101, (EM5): pp. 679–693.

[21] Hadjian, A.H. and Luco, J.E. (1977). *On the Importance of Layering on Impedance Functions.* Proceedings of the 6th WCEE, New Delhi, pp. 1675–1680.

[22] Israil, P.S.M. and Ahmad, S. (1989). *Dynamic Vertical Compliance of Strip Foundations in Layered Soils.* Earthquake Engineering and Structural Dynamics, 18(7): pp. 933–950.

[23] Ahmad, S. and Bharadwaj, A. (1991). *Horizontal Impedance of Embedded Strip Foundations in Layered Soil.* Journal of Geotechnical Engineering, ASCE, 117(7): pp. 1021–1041.

[24] Borja, R.I. and Wu, W.H. (1994). *Vibration of Foundations on Incompressible Soils with No Elastic Region.* Journal of Geotechnical Engineering. ASCE, 120(9): pp. 1570–1592.

[25] Wolf, J.P. (1996). *Foundation Vibration Analysis using Simple Physical Models.* PTR Prentice Hall, Inc., Englewood Cliffs, NJ 07632.

[26] Gazetas, G. and Stokoe, K.H., II (1991). *Free Vibration of Embedded Foundations: Theory Versus Experiment.* Journal of Geotechnical Engineering, ASCE, 117(9): pp. 1382–1401.

[27] Barkan, D. D. (1962). *Dynamics of Bases and Foundations.* McGraw-Hill Book Co., New York, N. Y.

[28] Fry, Z.B. (1963). *Development and Evaluation of Soil Bearing Capacity of Foundation Structures - Field Vibratory Tests Data.* Technical Report No. 1, US Army Engineer Waterways Experiment Station, Vicksburg, Mississippi.

[29] Beredugo, Y.O. and Novak, M. (1972). *Coupled Horizontal and Rocking Vibration of Embedded Footings.* Canadian Geotechnical Journal, 9(4): pp. 477–497.

[30] Stokoe, K.H. II and Richart, F.E. (1974). *Dynamic Response of Embedded Machine Foundations.* Journal of Geotechnical Engineering, ASCE, 100(4): pp. 427–447.

[31] Dobry, R., Gazetas, G. and Stokoe, K. H. II (1986). *Dynamic Response of Arbitrarily-shaped Foundations: Experimental Verification.* Journal of Geotechnical Engineering, ASCE, 112(2): pp. 136–154.

[32] Nii, Y. (1987). *Experimental Half Space Dynamic Stiffness.* Journal of Geotechnical Engineering, ASCE, 113(11), pp. 1359–1373.

[33] Crouse, C.B., Husmand, B., Luco, J.E. andWong, H. L. (1990). *Foundation Impedance Functions: Theory Versus Experiment*. Journal of Geotechnical Engineering, ASCE, 116(3); pp. 432–449.

[34] Baidya, D.K. and Muralikrishna, G. (2000). *Dynamic Response of Foundation on Finite Stratum – An Experimental Investigation.* Indian Geotechnical Journal, 30(4): pp. 327–350.

[35] Baidya, D.K. and Muralikirishna, G. (2001). *Investigation of Resonant Frequency and Amplitude of Vibrating Footing Resting on a Layered Soil System*. Geotechnical Testing Journal, ASTM, 24 (4): pp. 409–418.

[36] Raman, J. (1975). *Dynamic Response of Footing Soil System to Vertical Vibrations*. PhD Thesis, Indian Institute of Science, Bangalore, India.

[37] Moore, P.J. (1971). *Calculated and Observed Vibration Amplitudes*. Journal of Soil Mechanics and Foundation Engineering, ASCE, 97(1): pp. 141–158.

[38] Sridharan, A., Baidya, D.K. and Raju, D.M. (1992). *Prediction of Displacement Amplitude of Foundations at Any Frequency*. Indian Geotechnical Journal, 22(3): pp. 154–174.

[39] Baidya, D. K., Muralikrishna, G., and Pradhan, P.K. (2005). *Investigation of the Foundation Vibrations Resting on a Layered Soil System*. Accepted in Geotechnical and Geoenvironmental Journal, ASCE.

[40] Sulfi, A.M. (1993). *Vertical Vibrations Analysis of Rigid Footing on a Soil Layer with Rigid Base*. PhD Thesis, Texas Tech. University, USA.

[41] Baidya, D.K. (1998). *Settlement Computation – A Simplified Method.* Geotechnical Hazards, Proc. of the XI Danube Conference on SMGTE, Porec, Croatia.■