THE WIND MODEL SELECTION AND RELIABILITY ASSESSMENT OF A WIND ASSISTED UTILITY SYSTEM

C. Sharma* and D.D. Dalrymple**

ABSTRACT

This paper discusses the reliability modelling of generating systems including WECS. The methodology involved in the testing of hypothesized distributions is applied to demonstrate that the Weibull distribution in its hybrid form was found to be the most suitable model for representing empirical wind speed distributions in Trinidad and Tobago. Furthermore, the method of maximum likelihood parametric estimators produced a model which was far more accurate than either the method of moments or method of linear least squares estimators. A multistate model for WECS developed on the basis of the Weibull model, convolved with standard models for conventional generators were used to perform the reliability assessment of an isolated power system. The sensitivity tests conducted revealed that, from a reliability perspective the incentive to connect additional WECS to the electrical grid decreases as the penetration of WECS increases and the confidence in WECS is enhanced by better wind regimes.

1.0 INTRODUCTION

The substantial increases in fossil fuel prices subsequent to the oil crisis of 1973, together with the uncertainity as to the continuing availability of oil at or near required production levels served as an impetus which encouraged many nations to initiate substantial renewable energy R & D programmes. Wind energy has now emerged as one of the most promising if not the most promising of the renewable energy technologies.

Several mathematical models have been advocated in an effort to find known probability density functions which model wind speed distributions. Section 3 explores the reasonableness of four such models - the Raleigh, Log-normal, Gamma and Weibull for representing the actual wind speed data in Trinidad and Tobago. Having established the Weibull model as being the most accurate, the relative merits of three procedures for determining the Weibull parameters are considered.

Since wind energy, like all other renewable energy sources is intermittant, the fluctuating power output of WECS cannot be accurately represented by the two state Markov model. There are five methods for modelling and incorporating the stochastically varying power output of WECS in reliability studies. This paper focuses on the priniciples behind the development of one of these methods (the multistate unit approach) which essentially involves the combination of the performance characteristic of a selected wind turbine with the extrapolated Weibull modelled wind speed probability distribution to obtain a probabilistic profile of the loss of wind generation capacity due to (i) the inherent variability of the wind input and (ii) the FOR of the WECS. This profile (the multistate model) convolved with standard 2-state Markov models for conventional units was used to compute the LOLE reliability index using a program developed by the authors and data supplied by the Trinidad and Tobago Electricity Commission (T&TEC), under different scenarios. The results obtained are discussed and some conclusions are drawn regarding the reliability of wind-assisted utility systems.

2.0 FREQUENCY ANALYSIS OF THE WIND SPEED RECORDS

Trinidad and Tobago is not unlike many countries, in that, mean hourly wind speed data is recorded only at sparse locations: Piarco International airport, Crown Point International airport and Textel

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station at Matura. These measurements demonstrate a high degree of propriety for the synthesis of the multistate model with respect to both the type of anemometer employed and conformity to the ideal exposure as defined by the World Meteorological Organization [1].

Using the hourly wind speed data at these sites for 1988, monthly frequency tables were constructed, from which the monthly mean wind speeds were computed. These were used to identify three months for each site that represent maximum, typical average and minimum values of mean wind speed. (**Table 1**). The fitness of the wind speed models would be tested with references to these nine months. The following pertinent statistics were then obtained:

(i) values of the monthly standard deviations for the nine months (**Table2**)

(ii) probabilities of wind speeds occcuring within 1m/s intervals for (a) the nine months of Table 1 and (b) the year 1988

3.0 SELECTION OF THE WIND SPEED DISTRIBUTION MODEL

3.1 Probability Models

Several statistical models, notably the Rayleigh, Log-Normal, Gamma and Weibull have been suggested to model wind speed distributions [2].

The Rayleigh distribution, being a single parameter distribution and hence the easiest to use is given by the probability density function.

$$f(u) = \frac{\pi u}{2\mu^2} \exp\left[-\frac{\pi}{4}\left(\frac{u}{\mu}\right)^2\right] \tag{1}$$

The natural variability of the wind, however , tends to limit the need for greater sophistication of the following two-parameter models:

The Log-normal distribution given by the probability density function

$$f(u) = \frac{1}{\sqrt{2\pi.kl}} u^{-1} \exp\left[-\frac{(\ln u - cl)^2}{2.kl^2}\right] \tag{2}$$

The Gamma distribution given by the probability density function

$$f(u) = cg^{kg}\Gamma(kg)\ u^{kg-1} \exp\left[-\frac{u}{cg}\right] \tag{3}$$

The Weibull distribution given by the probability density function

$$f(u) = \frac{k}{c}\left(\frac{u}{c}\right)^{k-1} \exp\left[-\left(\frac{u^k}{c}\right)\right] \tag{4}$$

The equivalent cumulative distribution function of the Weibull is

$$F(u) = 1-\exp\left[-\left(\frac{u}{c}\right)^K\right] \tag{5}$$

3.2 Estimation Of Parameters

Applying the principle of the 'method of moments' [3] to equations 2-4, with respect to the first momemt about zero and the second moment about the mean, the following relations are obtained for the 2-parameter models:

The Log-normal - $u = \exp(cl + kl)$

$$S = \exp(2.cl + kl).(\exp\left[kl^2\right] - 1)$$

The Gamma - $u = kg.cg$
$S = kg.cg$

The Weibull - $u = c\Gamma(1 + 1/k)$

$$S^2 = c^2\left[\Gamma(1 + 2/k) - \Gamma^2(1 + 1/k)\right]$$

Thus, the simultaneous solution of each pair of relation using the computed monthly means and variances gives the shape scale parameters of the mathematically modelled wind speed distributions for the designated nine months (see **Table 2**).

3.3 Reasonableness of the Models

When a theoretical distribution has been assumed to represent a physical phenomenon, two different approaches can be employed to assess its fitness to the actual data.

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When a theoretical distribution has been assumed to represent a physical phenomenon, two different approaches can be employed to assess its fitness to the actual data.

CLASS	CATEGORY	MONTH: PIARCO AIRPORT	MONTH: CROWN POINT AIRPORT	MONTH: MATURA
I	Maximum value of mean wind speed	MAY	MAY	MAY
II	Typical average wind speed	JANUARY	JANUARY	JULY
III	Minimum value of mean wind speed	AUGUST	AUGUST	JANUARY

Table 1: Months representing Maximum (Class I), Typical Average (Class II) and Minimum (Class III) values of Mean Wind Speed

	SITE	MATURA			PIARCO AIRPORT			CROWN POINT AIRPORT		
	Month	May	July	January	May	January	August	May	January	August
u/ms^{-1}		6.014	3.619	1.846	4.385	2.984	1.510	4.100	2.530	1.760
s/ms^{-1}		1.557	1.768	1.616	2.162	2.619	1.885	1.499	1.841	1.729
Rayleigh	u/ms^{-1}	6.014	3.619	1.846	4.385	2.984	1.510	4.100	2.530	1.760
log -	kl	0.255	0.463	.0.754	0.467	0.756	0.969	0.354	0.652	0.822
normal	cl	1.762	1.179	0.329	1,369	0.808	-0.058	1.348	0.716	0.228
Gamma	kg	14.919	4.190	1.305	4.14	1.298	0.642	7.482	1.889	1.036
	cg/ms^{-1}	0.403	0.864	1.415	1.066	2.299	2.353	0.548	1.340	1.699
Weibull	k	4.339	2.177	1.156	2.155	1.152	0.786	2.982	1.412	1.020
	c/ms^{-1}	6.604	4.087	1.942	4.951	3.137	1.316	4.593	2.780	1.774

Table 2: Means, Standard Deviations and Moment Estimators of Parameters for Classes I, II and III Months

The first technique, illustrated in **Figures 1-3**, involves the graphic comparison of f(u) with the wind speed frequency curve. The wind speed curve is simply a plot of the probabilities of wind speeds occuring within 1m/s intrervals vs the wind speed interval medians. This method is frequently used in wind power analysis, albeit its subjective nature. Examination of the plots reveal (i) the greater versatility afforded by the 2-parameter models results in an improvement to the fit and (ii) the fit of the Weibull is better than the Gamma and Log-normal, especially in the tail of the distributions.

Statistical tests, which constitute the other technique, are more objective and can be useful in supplementing the former method. Referring to **Table 3** which gives the computed Chi-squared test statistic X^2c[4] for the months under consideration, it is concluded in every case that the empirical wind speed data contradicts the assumption of all four models for wind speed probability distribution since $X^2{}_c >> C_{0.999,F}$; but with such large large quantities of data this is only to be expected. In spite of this however, the Chi-squared goodness-of-fit test has substantiated the findings elucidated by the former technique i.e the Weibull is superior to the other three models because (i) X c for the Weibull is the least in five out of nine cases and (ii) the summation of $(o_i-e_i)^2/e_i$ in the Weibull is less than the corresponding summation of the Rayleigh (its closest rival) for three out of the remaining four cases (**Table 4**).

3.4 Two other Methods for the Parametric Estimation of the Weibull

Taking logarithms twice on equation 5 transforms it to the linear form $y = ax + b$ by the relations $x_i = \ln u_i$ and $y_i = \ln\left[-\ln(1-F(u_i))\right]$. Using a frequency table, the best-fit linear coefficients a + b can be found by the weighted least squares process [5]. The Weibull parameters are related to the linear coefficients by

$$k = a \quad \text{and} \quad c = \exp\left[-b/k\right]$$

Applying the logic of the method of maximum likelihood [4] to the Weibull distribution , the likelihood parametric estimators for a frequency table are obtained from the equations reported in reference 6. Since the solution of these equations require about 20-40 iterations for successful convergence [7], an approach, incorporating the Newton-Raphson second method, was contrived by the authors which was more economical in computer time [1].

The 'method of linear least squares' and 'method of maximum likelihood' estimators for the nine months are listed in **Table 5**.

3.5 Accuracy of the Three Methods

Assuming an atmospheric pressure of 101.3 N/m^2 and a temperature of 303K, the actual average wind power density for a frequency table of wind speed is [1]

$$\overline{Pwa} = 0.5817 \sum_{i=1}^{w} p(ui)u_i^3 \ \mathrm{W/m^2} \qquad (6)$$

The theoretical average power density based on the Weibull model is

$$\overline{Pwt} = 0.5817c^3\ \Gamma\left(1+3/k\right)\ \mathrm{W/m^2} \qquad (7)$$

The magnitude in the disparity betwen equations (6) and (7) is indicative of the accuracy of the Weibull models obtained using the methods of moments least squares and maximum likelihood. These values are tabulated in **Table 5**. On examination of **Table 5**, it is inferred that the Weibull parameters are best obtained using the maximum likelihood method because the resulting Weibull model most accurately predicts the average available power in the wind.

3.6 Weibull Hybrid Density Function

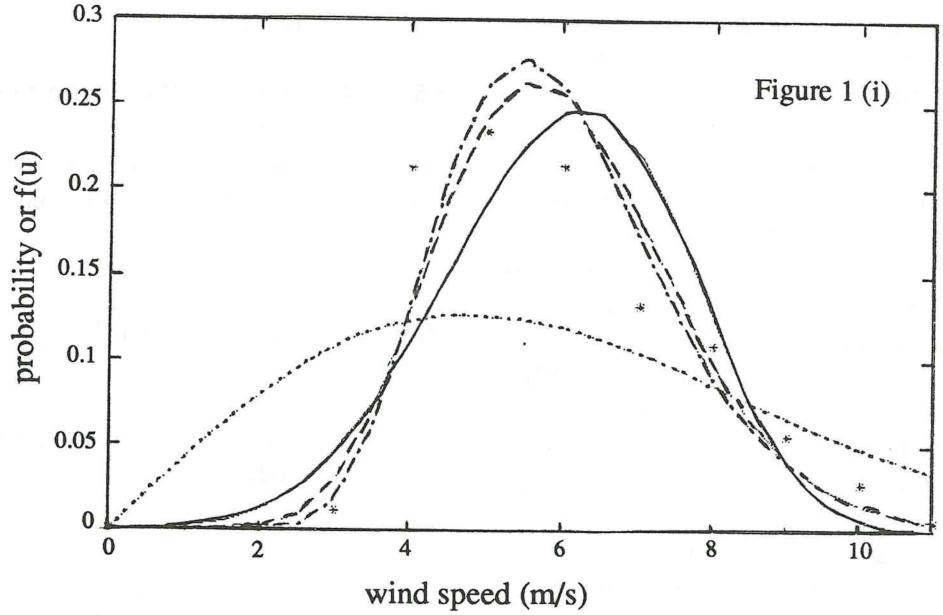
A limitation with the Weibull model, encountered thus far, is its inability to properly account for calms since equation 4 vanishes at u=o. A way of reducing this problem is through the use of the hybrid density function of the form [6].

$$f_h(u) = p(u=o)\partial(u) + [1-p(u=o)]f(u) \qquad (8)$$

where f(u) is equation 4 with $k \rightarrow k_h$ and $c \rightarrow c_h$ are estimated from the wind speed frequency table after calms are removed. The corresponding theoretical average power density is

$$\overline{Pwt} = 0.5817[1-p(u=0)]c_h^3\Gamma(1+3/k_h) \qquad (9)$$

Table 6 summarizes the pertinent quantities for classes I,II,and III months. A comparison of the priate



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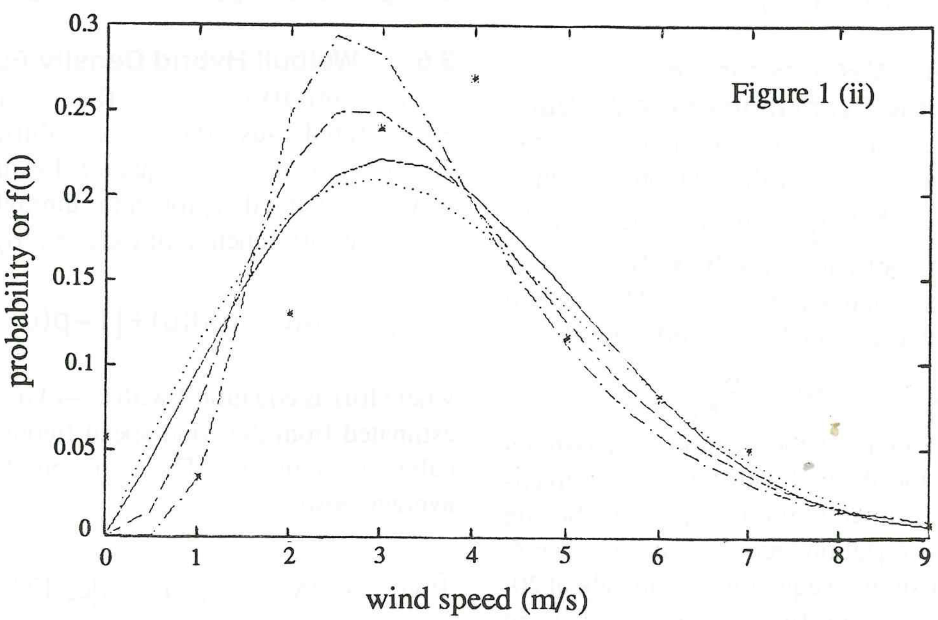

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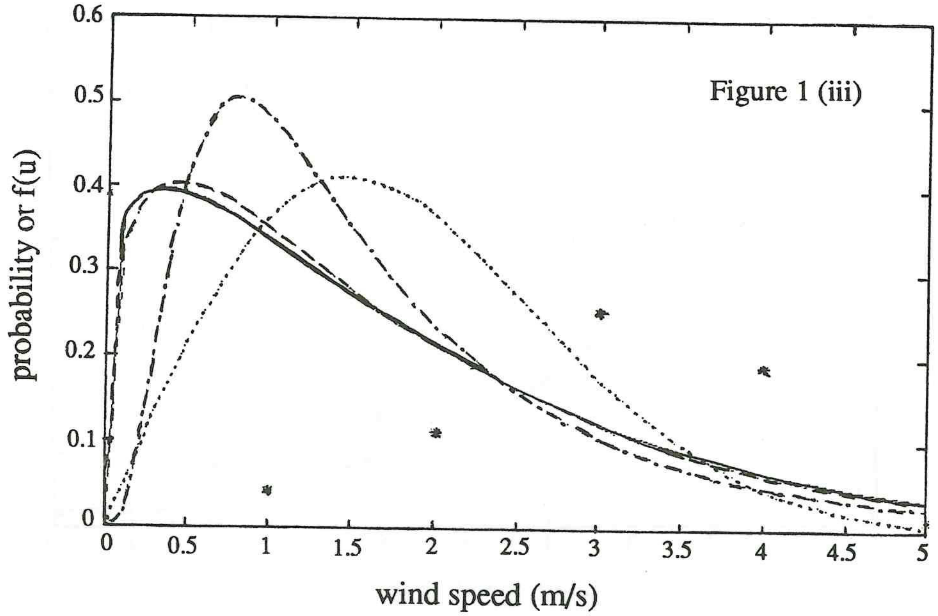

Actual wind data and probability density functions for Matura (i) May, (ii) July, (iii) January

——	Weibull
– –	Gamma
–·–	Log-normal
· · · ·	Raleigh
*	Actual wind data

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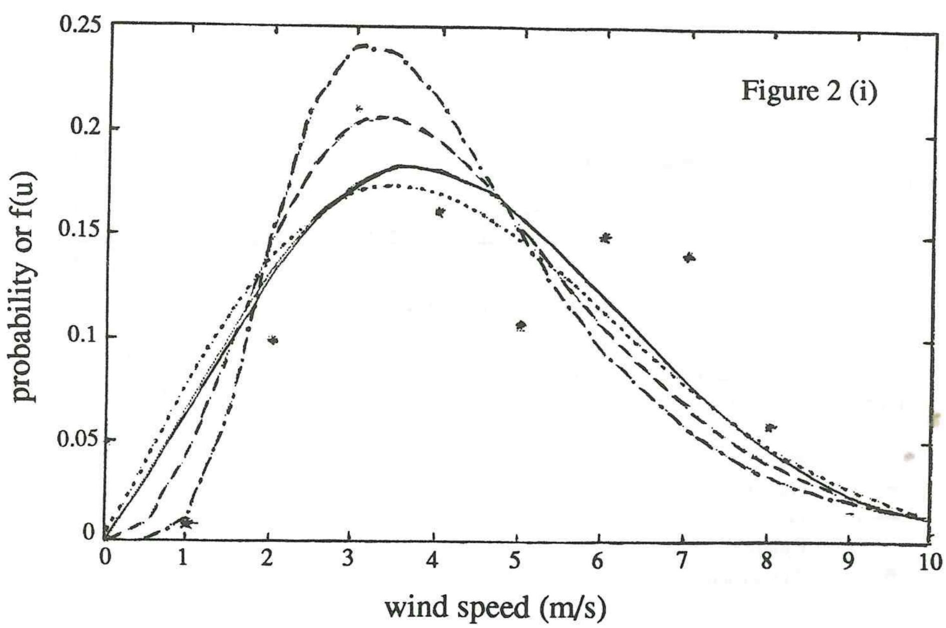

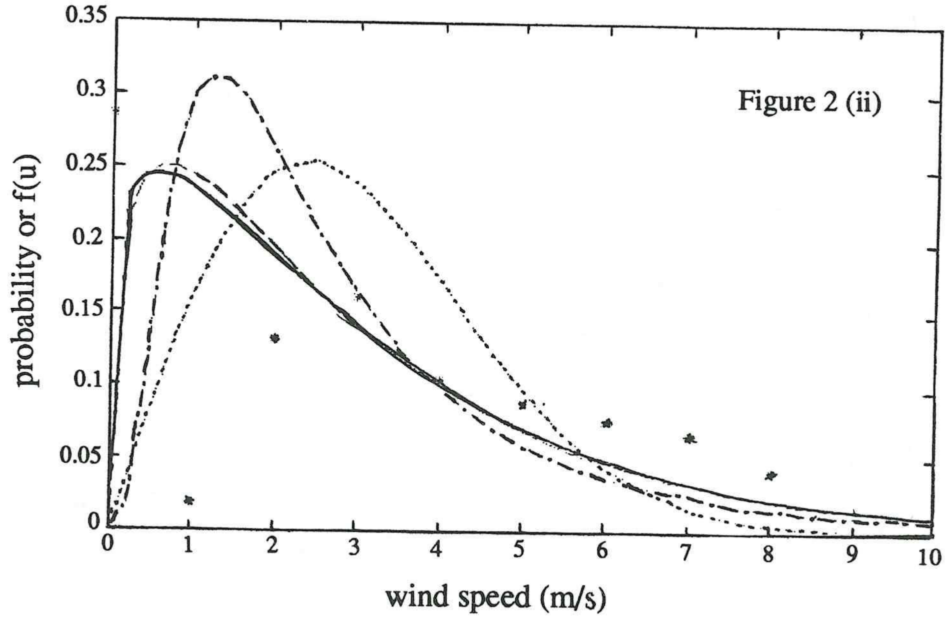

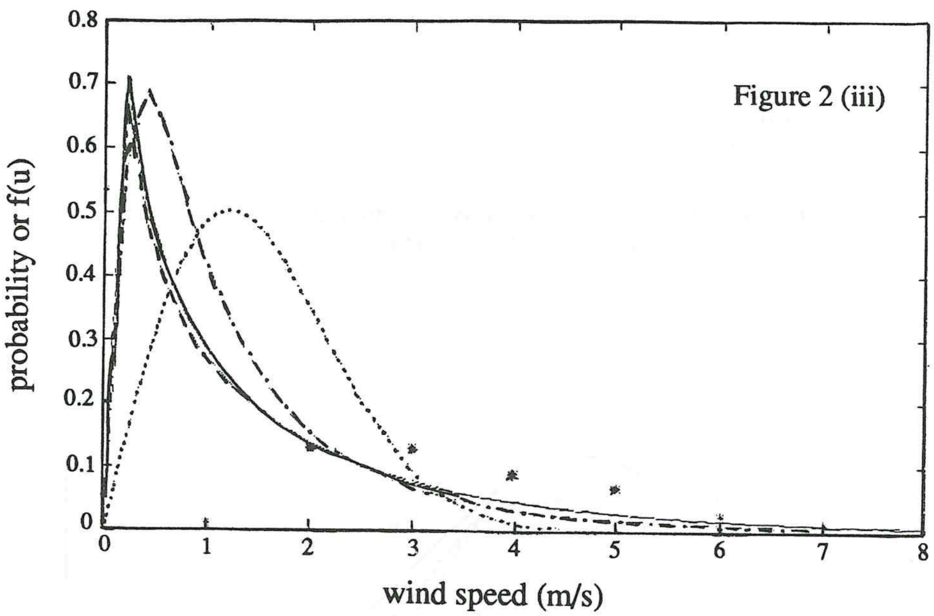

Actual wind data and probability density functions for Piarco
(i) May, (ii) January, (iii) August

——— Weibull
– – — Gamma
—· — Log-normal
········ Raleigh
* Actual wind data

Figures 2 (i), (ii) & (iii): Actual Wind Data and Probability Density Functions for Piarco.

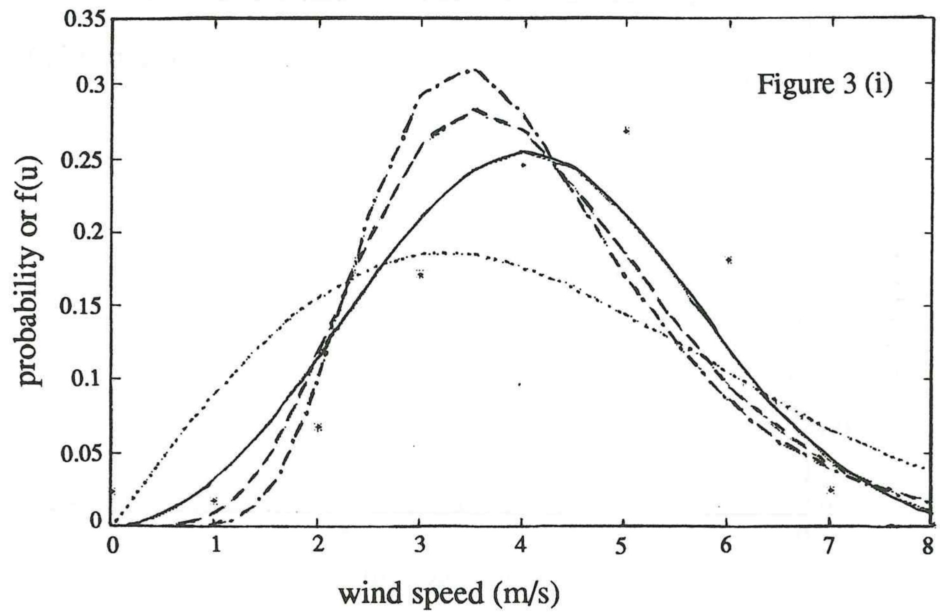
Figure 3 (i)
probability or f(u)
wind speed (m/s)
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
6
7
8

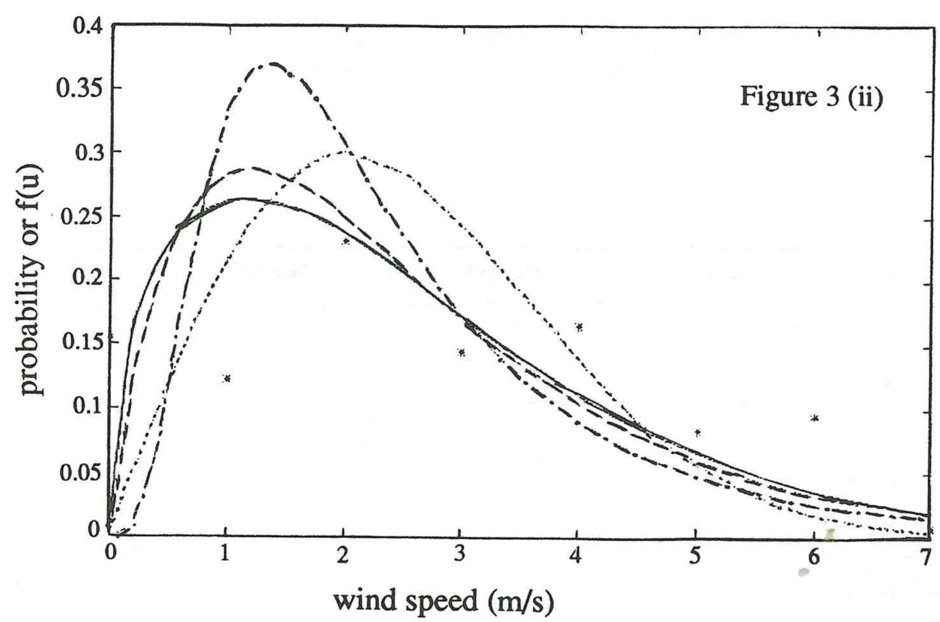
Figure 3 (ii)
probability or f(u)
wind speed (m/s)
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
6
7

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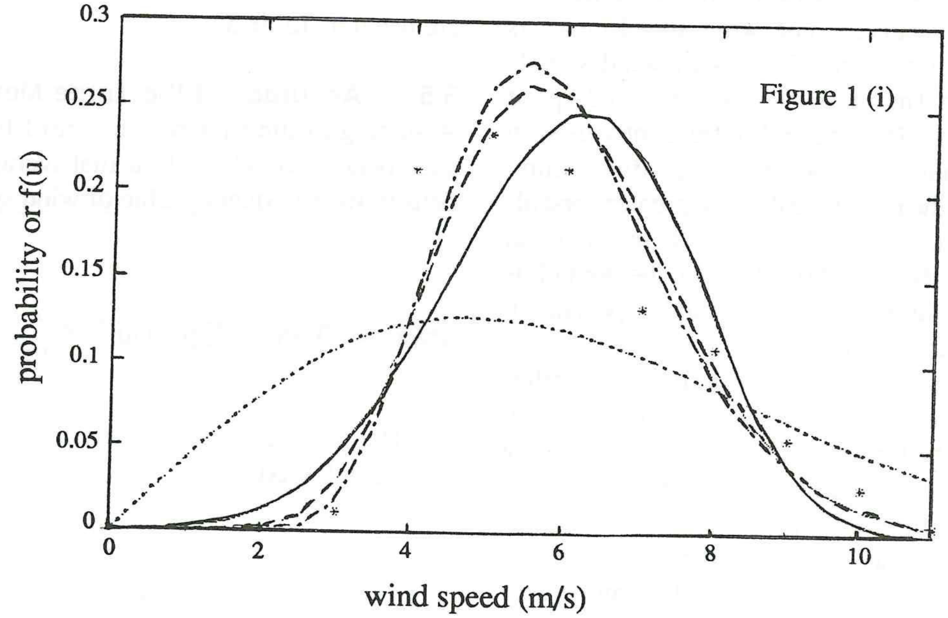

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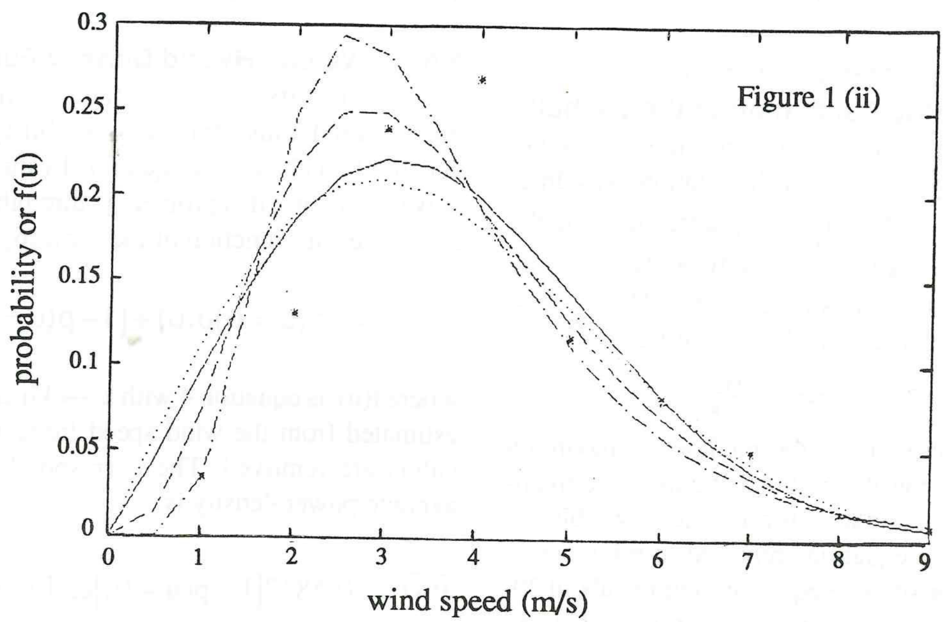
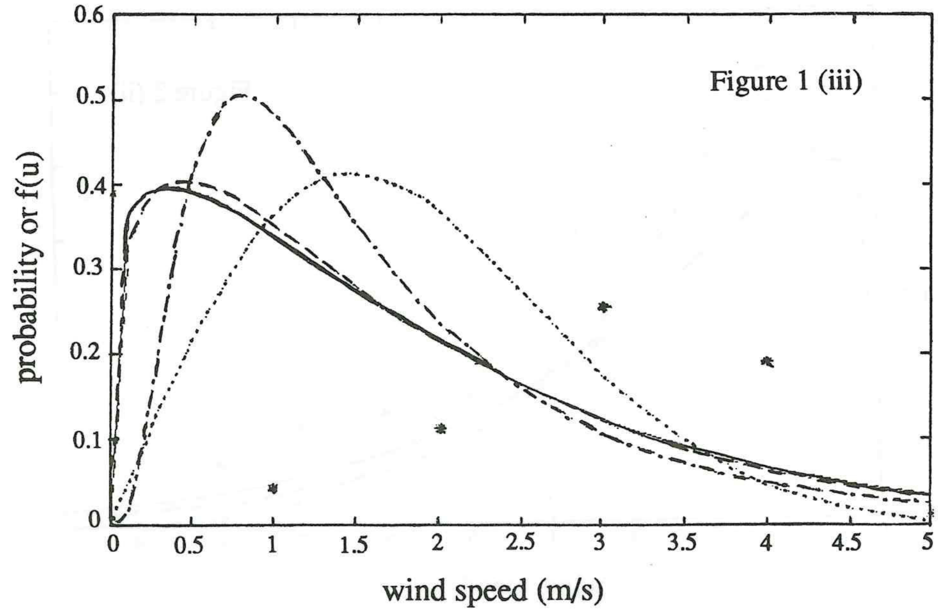


Figure 1 (ii)



Actual wind data and probability density functions for Matura (i) May, (ii) July, (iii) January

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*	Actual wind data

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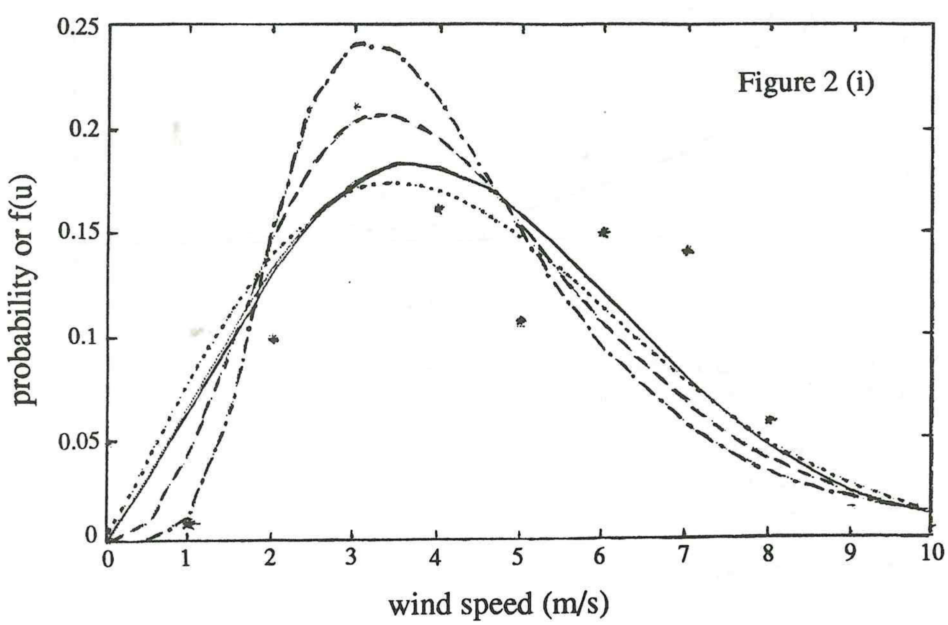

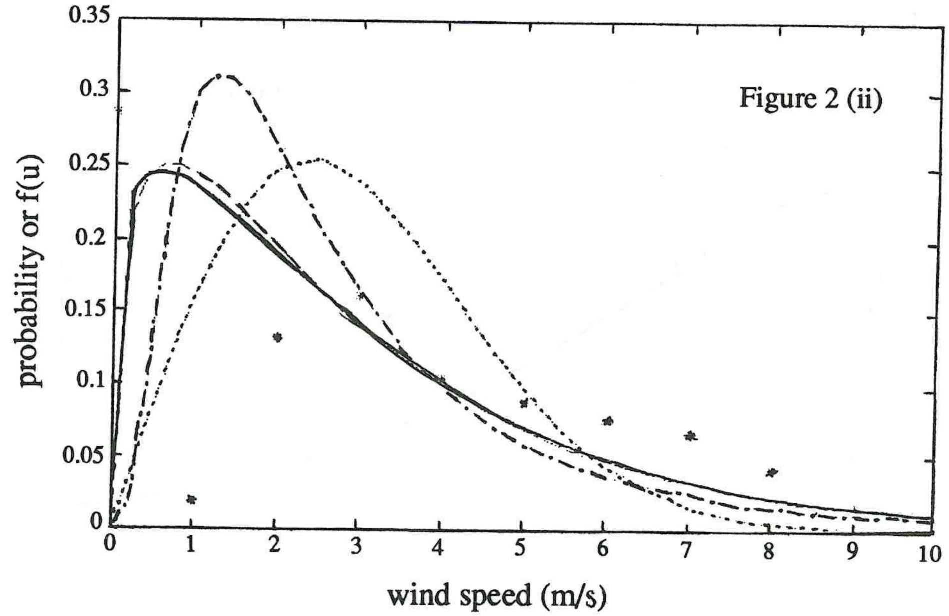

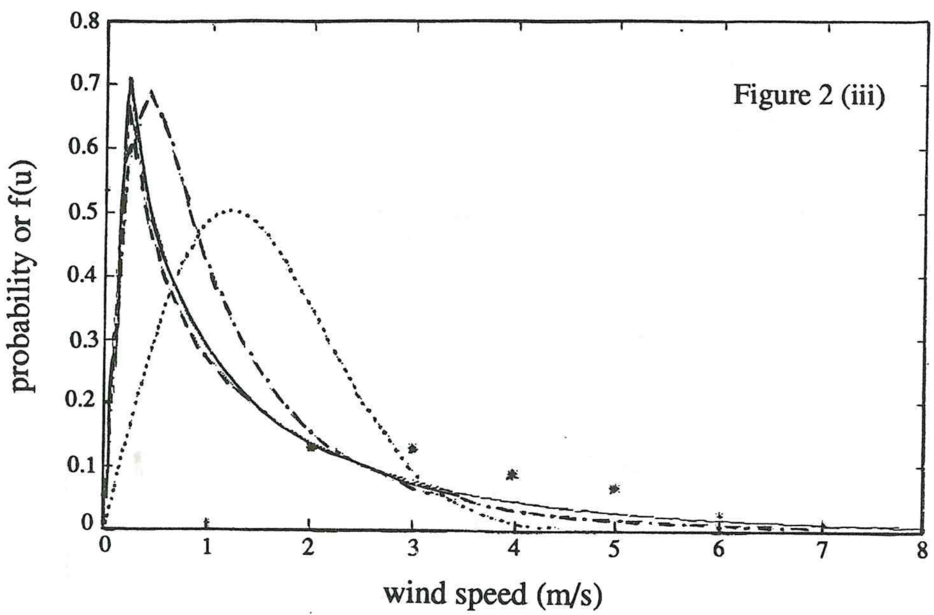

Actual wind data and probability density functions for Piarco
(i) May, (ii) January, (iii) August

——— Weibull
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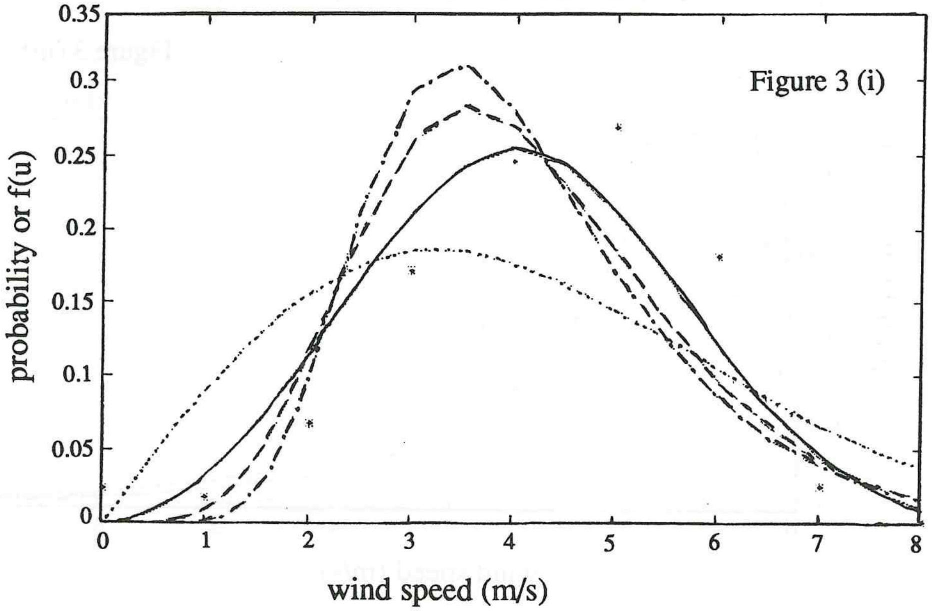
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wind speed (m/s)

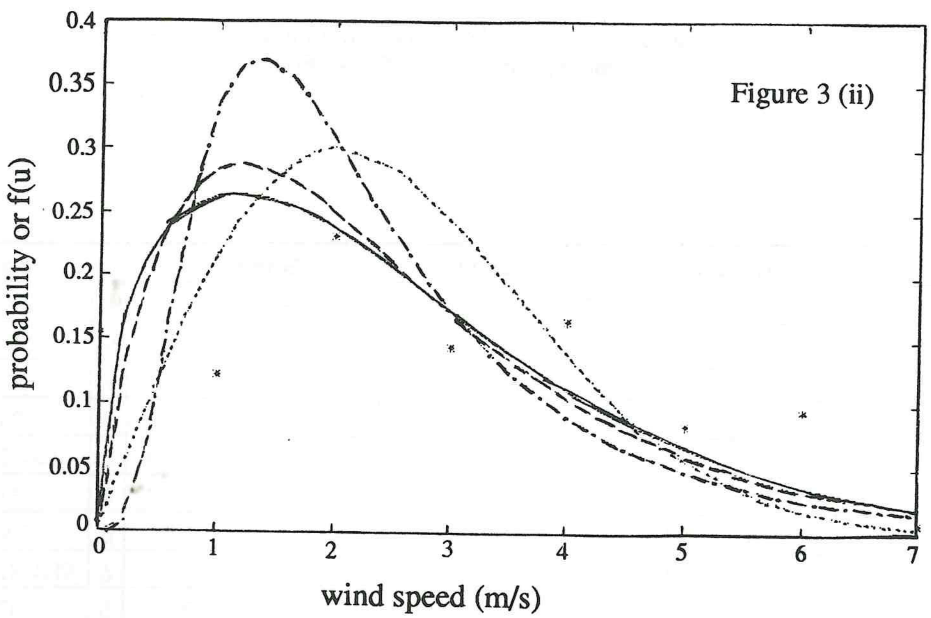
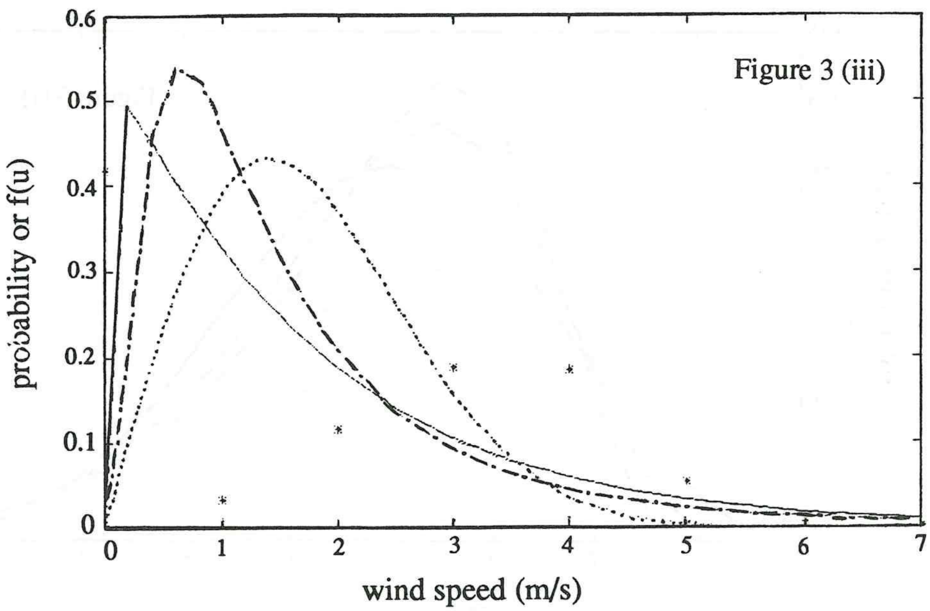


Figure 3 (ii)
probability or f(u)
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
6
7
wind speed (m/s)



Actual wind data and probability density functions for Matura (i) May, (ii) July, (iii) January

——— Weibull
– – – – Gamma
— · — Log-normal
.... Raleigh
* Actual wind data

Figures 3 (i), (ii) & (iii): Actual Wind Data and Probability Density Functions for Crown Point.

	Rayleigh			Weibull			Gamma			Log-normal		
Site, Month	F	$X_c{}^2$	$C_{0.999,F}$	F	$X_c{}^2$	$C_{0.999,F}$	F	$X_c{}^2$	$C_{0.999,F}$	F	$X_c{}^2$	$C_{0.999,F}$
Matura,May	10	368	29.6	9	220	27.9	9	76	27.9	9	66	27.9
Matura,May	8	96	26.1	7	232	24.3	7	1202	24.3	7	92505	24.3
Matura,May	4	2105	18.5	3	670	16.3	3	711	16.3	3	1523	16.3
Piarco,May.	9	233	27.9	8	290	26.1	8	1800	26.1	8	348481	26.1
Piarco,Jan.	9	3132	27.9	8	406	25.1	8	461	25.1	8	2481	26.1
Piarco,Aug.	7	606168	24.3	6	355	22.5	6	364	22.5	6	738	22.5
C.Point,May	7	268	24.3	6	440	22.5	6	24176	22.5	6	98864216	22.5
C.Point,Jan.	6	617	22.5	5	174	20.5	5	239	20.5	5	1147	20.5
C.Point,Aug.	6	2693	22.5	5	555	20.5	5	558	20.5	5	1159	20.5

Table 3: Chi-squared Test for Relative Goodness-of-fit between the Four Models; Classes I, II and III Months.

	$(o_i-e_i)^2/e_i$ m/s					
Interval	Matura, July		Piarco, May		Crown Point, May	
m/s	Weibull	Rayleigh	Weibull	Rayleigh	Weibull	Rayleigh
2.5 - 3.5	1	4	6	8	5	0
3.5 - 4.5	18	29	1	0	0	20
4.5 5.5	4	2	13	8	12	77
5.5 - 6.5	0	0	5	9	21	41
6.5 - 7.5	2	1	33	36	7	19
7.5 - 8.5	0	1	2	1	6	25
8.5 - 9.5	0	1	3	4		
9.5 - 10.5			3	5		
	Σ25	Σ38	Σ66	Σ71	Σ51	Σ182

Table 4: Relative Goodness-of-fit between the Weibull and the Rayleigh functions in the Tail of the Distribution.

		Method of Moments	Linear Least Squares			Maximum Likelihood		
SITE MONTH	Pwa w/m^2	Pwt	k	c m/s	Pwt w/m^2	k	c m/s	Pwt w/m^2
Matura, May	153.90	151.97	2.771	7.217	226.92	3.981	6.619	155.17
Matura, July	48.06	48.63	2.180	3.915	42.69	2.206	4.100	48.53
Matura, January	12.12	15.75	1.430	1.999	10.19	1.332	2.191	15.63
Piarco, May	84.87	87.24	2.057	4.449	66.16	2.186	4.966	86.93
Piarco, January	56.61	67.06	1.144	2.883	53.15	1.222	3.342	68.69
Piarco, August	15.08	24.24	1.041	1.581	11.91	1.112	1.862	15.63
Crown Point, May	52.75	56.51	2.657	4.078	41.88	3.139	4.582	54.96
Crown Point, January	25.70	28.13	1.810	2.217	9.47	1.528	2.902	27.49
Crown Point, August	13.42	18.11	1.317	1.809	9.03	1.284	2.136	15.80

Table 5: Comparison of the Average Wind Power Densities (Actual with Theoretical) for the Three Periods.

SITE , MONTH	P(u = 0)	K_h	C_h m/s	Pwt w/m^2
Matura ,May	0.0000	3.981	6.619	155.17
" ,June	0.0579	1.618	4.333	47.74
" ,January	0.3887	3.946	3.333	12.13
Piarco ,May	0.0484	2.551	5.206	84.95
" ,January	0.2863	2.088	4.740	56.27
" ,August	0.5349	2.480	3.673	14.85
C. Point,May	0.0236	3.493.	4.671	54.89
" ,January	0.1533	1.969	3.387	25.87
" ,August	0.4170	2.906	3.392	13.42

Table 6: The Parameters and the Predicted Mean Power Densities of the Hybrid Weibull Model for Classes I, II and III Months.

appropriate columns of **Tables 5** and **6** reveals excellent agreement between the actual and hybrid Weibull models predicted wind power densities. The hybrid Weibull distribution is thus the most accurate model for representing the wind speed distribution in Trinidad and Tobago, a recognition further confirmed graphically in **Figure 4.**

4.0 OPTIMAL SELECTION OF THE WECS

It was decided to 'install' the WECS at Matura because a comparative study of the three sites long term average wind power confirmed that Matura was best suited for exploitation of the wind resource [1].

4.1 Typical Performance Characteristic of a Large WECS

The salient features of a large WECS performance characteristics depicted in **Figure 5** are :

(i) a minimum or 'cut-in' wind speed, u_C, is required before electrical power is supplied,

(ii) a maximum or 'furling' wind speed, u_F at which the turbine is shut down to prevent structural damage,

(iii) a 'rated' wind speed, u at which the turbine first produces its designed output, P_{eR},

(iv) the machine produces its rated output for wind speeds between u_R and u_F, independent of wind speed in between,

(v) the relationship between the electrical output and wind speed is non-linear in the speed range ($u_C<u\leq u_R$) due to the combined effects of aeroturbine, transmission and generator characteristics. In the absence of an actual power function curve from the manufacturer, it is necessaary to approximate the performance characteristic of a WECS in this partial power range by one of several mathematical models. Thus, the complete model is accepted to be [8]

$$Pe(u)\begin{cases} 0 & u \leq u_c \\ Pe(u) = 2 + bu^{kh} & (u_c < u \leq u_R) \\ PeR & u_R \leq u \leq u_F \\ 0 & u > u_F \end{cases} \quad (10)$$

4.2 Extrapolation of Wind Speed Distribution with Height

Since the performance estimates (including the probabilistic profile required in this study) can only be derived when the wind speed probability distribution referenced to the hub-height is integrated with the power function given in terms of wind speed at hub-height, it is necessary to have a statistical description of the variation of wind speed with height. In this paper, the following results developed by Justus and Mikhail are used for the extrapolation of the hybrid Weibull parameters.

$$\left.\begin{aligned} \frac{k_{h2}}{k_h} &= \frac{1-0.881\ \ln(z_1/10)}{1-0.881\ \ln(z_2/10)} \\ \frac{c_{h2}}{c_h} &= \left(\frac{z_2}{z_1}\right)^{np} \\ np &= \frac{0.37-0.0881\ \ln c_h}{1-0.0881\ \ln(z_1/10)} \end{aligned}\right\} \quad (11)$$

4.3 Optimal WECS for Matura

For a WECS to be economically viable, the system must be optimally selected from several commercial options. Two techniques will now be considered for selecting a WECS for Matura. Since the installed capacity of T&TEC is in the giga watt range, the selective process is confined to WECS in the MW range.

Technique 1

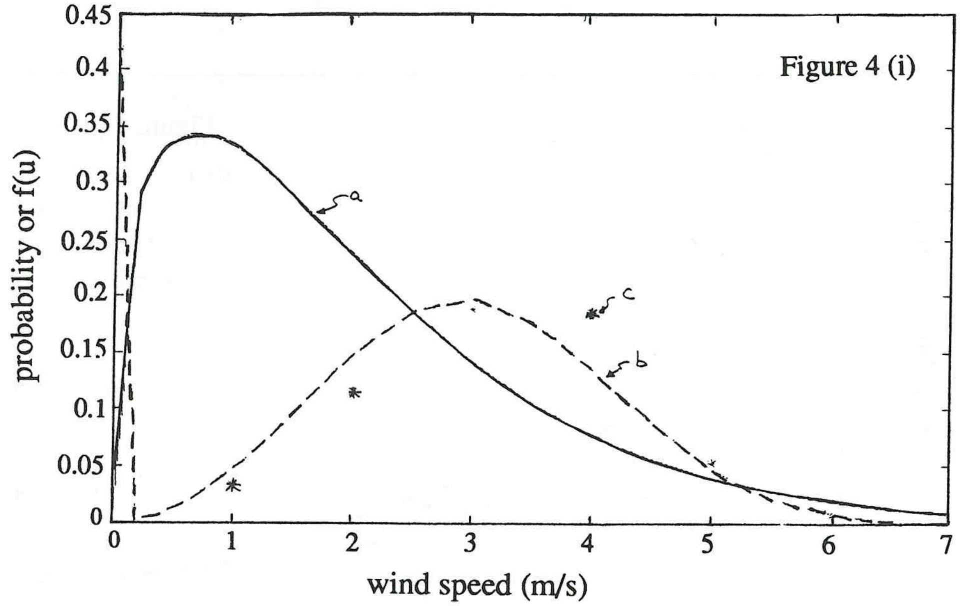
For a Weibull modelled wind speed distribution

$$\overline{Pwt} = \frac{1}{2}\rho\int_{o}^{\infty} u^3 f(u)du \quad (12)$$

A plot of u^3 f(u) illustrates a fundamental feature of the function u^3 f(u) i.e. it has a maximum at a wind speed, u_{mp}, which can be shown to be related to the Weibull parameters by

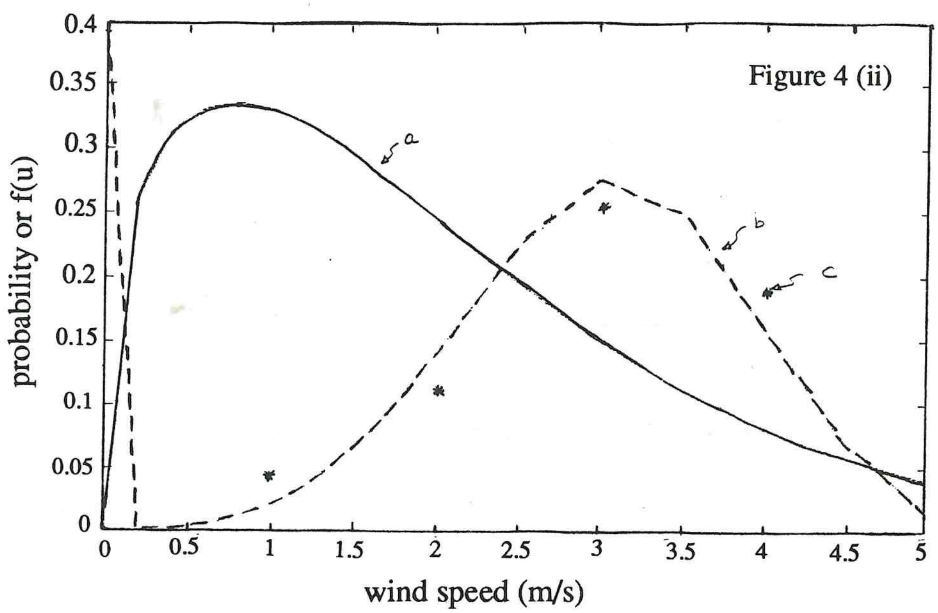
$$U_{mp} = C_{h2}\left(\frac{k_{h2}+2}{k_{h2}}\right)^{1/k_{h2}} \quad (13)$$

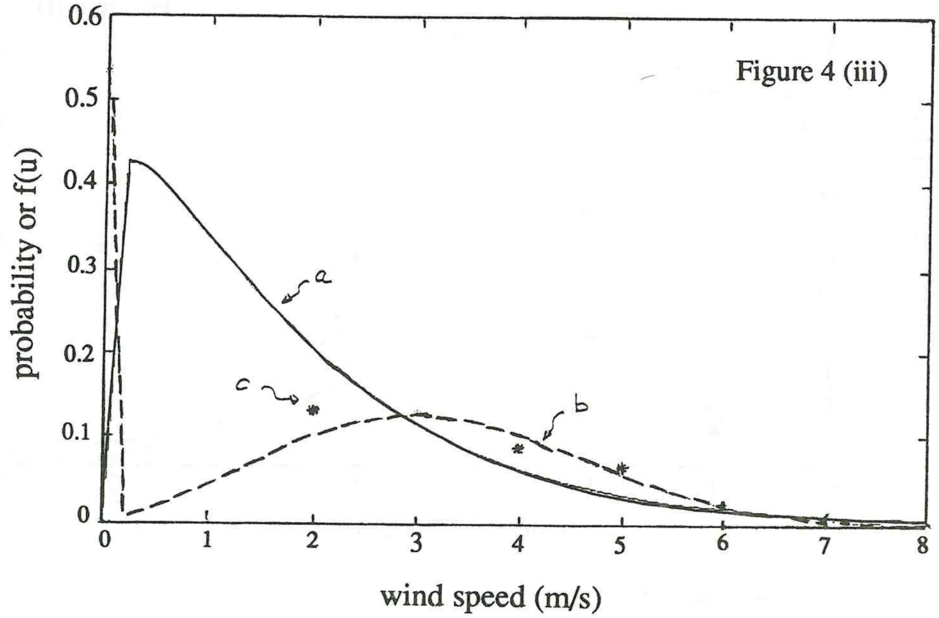
A commercial wind turbine should be selected according to the criteria that is as close as possible to u_{mp} [5].



Actual wind data, hybrid Weibull and standard Weibull (non hybrid) density functions for Class III months
(i) Matura, January (ii) Piarco, August and (iii) Crown Point, August

a —— Standard Weibull
b – – Hybrid Weibull
c * Actual wind data





Figures 4 (i), (ii) & (iii): Actual Wind Data, Hybrid Weibull and Standard Weibull.

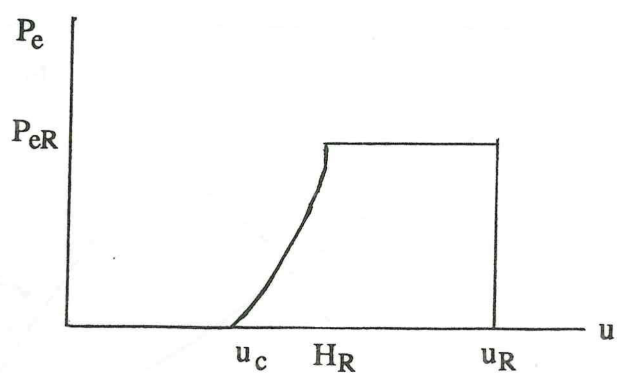


Figure 5: Performance Characteristic of a Large WECS.

The specifications of three WECS are listed in **Table 7**[9]. The extrapolated annual hybrid Weibull parameters for Matura referred to the hubheights are calculated using equation 11. Thus U_{mp} at the hubheights, are calculated from equation 13. These quantities are tabulated in **Table 8**. Comparing column 5 of **Table 7** with column 4 of **Table 8**; it is observed that U is closest to in the case of Growian II. Thus, this turbine is the best choice.

Technique 2

The average power output which is to be expected from a WECS is

$$\overline{P_e} = \int_0^\infty P_e(u)f(u)du \quad w \tag{14}$$

Substituting equation 8 and 10 into equation 14 and defining the concept of normalized power [5], the following relation is obtained.

$$P_N = (CF)\,(U_R / C_{H2})^3 \tag{15}$$

where

$$CF = \{1 - p(u=0)\} \quad X$$

$$X = \frac{\exp\left[-\left(\frac{uc}{c_{h2}}\right)^{k_{h2}}\right] - \exp\left[-\left(\frac{u_R}{c_{h2}}\right)\right]^{k_{h2}}}{\left(\frac{u_R}{c_{h2}}\right)^{k_{h2}} - \left(\frac{u_R}{c_{h2}}\right)} - \exp\left[-\left(\frac{u^R}{c_{h2}}\right)^{k_{h2}}\right] \tag{16}$$

Families of curves, pertaining to equation 15 can now be plotted to show the optimum ratio of U_R/C_{h2} (normalized rated speed) to give the maximum annual energy production at a site.

For a particular commercial WECS, the ratios of U_R/U_C and U_f/U_R are specific and hence CF (equation 16) is a function of the variable $\left(U_R/C_{h_2}\right)^{k_{h2}}$

Plots of Pn vs (U_R/C_{h2}) for Growian II (equation 15), are given in **Figure 6**. These curves show clearly the essential ratios of U_R/C_{h2}) to give the best energy recovery for different sites (or different k_{h2}) Thus, since the curve for Matura (k_{h2}=3.397) peaks at a normalized rated speed of 1.2 and since C_{h2} equals 8.896, the optimum value of U_R is determined to be 8.896 x 1.2 = 10.68. The optimal rated speed for WTS-4 and Tvind were similarily obtained (see **Table 9**).

Comparing column 5 of **Table 7** with column 3 of **Table 9**, it is again observed that U_R is closest to the optimal U_R in the case in Growian II.

The Growian II is therefore selected to be 'installed' at Matura since its designed specification matches very closely to the wind regime for optimum performance.

5.0 RELIABILITY ANALYSIS

The basic probabilistic approach for calculating the LOLE reliabiltiy index for any generating system can be summarized by the following steps [10]:

(i) select the appropriate model for each generating unit, depending on the charateristic of each unit,

(ii) develop a capacity model for the generating system using the models of each unit,

(iii) develop a suitable load model from the estimated data, and

(iv) combine the model for the generating system with the load model to obtain the LOLE of the system.

5.1 Models for Generating Units Conventional Units

In this analysis, all conventional generating units are treated as repairable units which have only two operational model- 2-state Markov models [11]. The unit of unavailability is the well known FOR.

WECS

Each WECS is represented by a multistate model. In this concept, the available wind power is described by a number of capacity states which correspond to various levels of wind power production and their associated probabilities [12]. Developing the multistate model for the WECS requires the consideration of the following three factors which affects the WECS electrical power output:

(i) the wind speed probability distribution,

(ii) the performance characteristic of the WECS, and

(iii) the FOR of the WECS

NAME	P_{eR} MW	Z_2 m	U_C m/s	U_R m/s	U_F m/s
Growian II	5	120	6.6	11.25	20
WTS -4	4	80	7.1	16.20	27
Tvind	2	53	5.0	14.50	20

Table 7: Specifications of Three WECS

NAME	K_{h2}	C_{h2} m/s	U_{mp} m/s
Growian II	3.397	8.896	10.19
WTS -4	3.248	8.113	9.40
Tvind	3.110	7.388	8.67

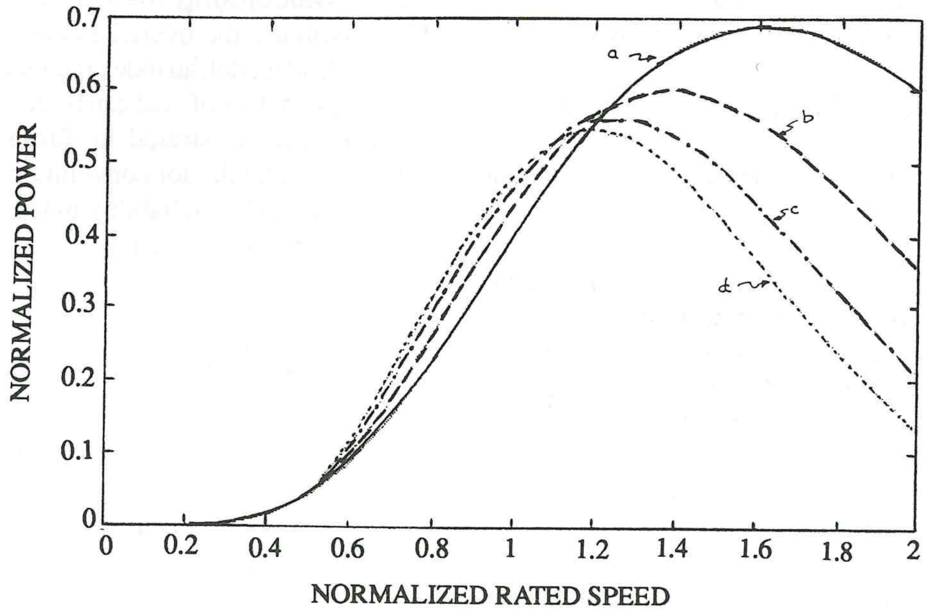
Table 8: Hybrid Weibull Parameters and Maximum Available Power Wind Speed referred to the Hubheights.

NAME	Optimum U / Ch	Optimum U
Growian II	1.20	10.68
WTS -4	1.30	10.55
Tvind	1.45	10.71

Table 9: Optimal Values of the Normalized Rated Speed and Rated Speed for the Three WECS.

LOAD LEVEL (PU)	FREQUENCY (DAYS)
1.0	4
0.9770	4
0.9594	4
0.9425	4
0.9228	4
0.8916	4
0.8601	4

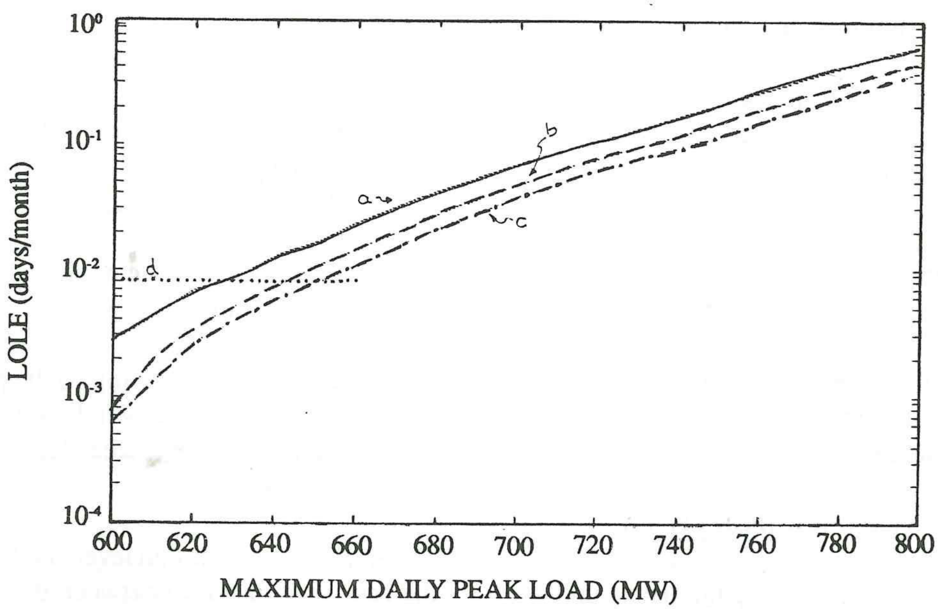
Table 10: Markov Daily Peak Load Model, T&TEC's System, 28 days Period



Normalized power versus Normalized rated speed for Growian II

a —— $k_{h2} = 2$
b — — $k_{h2} = 2.5$
c —·— $k_{h2} = 3$
d ······ $k_{h2} = 3.397$

Figure 6: Normalized Power Versus Normalized Rated Speed for Growian II.



LOLE vs maximum daily peak load for T&TEC system without and with WECS and with 25 MW conventional unit, May 1988.

a —— Base generating system (1052 MW)
b — — Base generating system plus 5 NG, (Pen = 2.32%)
c —•— Base generating system plus 25 MW, 0.075 for conventional unit.
d ······ Design LOLE level (0.0083 days/month)

Figure 7: LOLE vs. Max. Daily Peak Load for T&TEC System.

The approach used in this paper for the building of the multistate model involves the following steps:

(1) obtain c_h and k_h for the month considered,

(2) compute c_{h2} and k_{h2} at the hubheight of Growian II,

(3) elect the number of derated operating capacity states one wishes to represent the variation in electrical power output between u_c and u_r and hence evaluate the wind speed intervals corresponding to each derated capacity state,

(4) the exact probability associated with each derated state is calculated,

(5) the required operating capacity related to each derated state is obtained by combining the Growian II performances characteristics over the speed range

$$u_c<u<u_r(Pe(6.6<u<11.25)=0.006\,u^{3.397}-0.9765) \quad (16)$$

and the hybrid Weibull wind speed model,

(6) the rated operating capacity and zero operating capacity probabilities are computed,

(7) the probabilities calculated in steps 4 and 6 are modified to reflect the effect of an assumed FOR = 5% for Growian II.

5.2 Capacity Model for the Generating System

The capacity outages of the CCOPT (which provide indications of all probable deficiencies) have a discrete cumulative probability distribution whch can be evaluated using the following recursive relation for conventional unit addition [13].

$$F_a(PX_i)=(1-FOR_j)\,F_b(PX_i)+(FOR_j)\,F_b(PX_i-C_j) \quad (17)$$

The corresponding relation for a multistate unit addition is

$$F_a(PX_i)=\sum_{i=1}^{q} p(x_i)F_b(PX_i-x_i) \quad (18)$$

A transaction value of 10^{-4} is used in the compilation of the CCOPT.

5.3 Calculating the LOLE Index

By convolving the overall system's CCOPT with a suitable load model, an index representing the expected risk of system loss of load can be derived. The Markov chain model, illustrated in **Table 10**, is the most suitable load model for convolution with the CCOPT model. The LOLE reliability index is computed from the following summation

$$LOLE=\sum_{i=1}^{q} p(re_i) \quad (19)$$

where re_i = installed capacity minus peak load on day i

$p(re_i)$ = probability that re_i is less than x_i on day i

6.0 SENSITIVITY TESTS, RESULTS AND DISCUSSION

In order to appreciate the manner in which the basic parameters that are related to WECS may influence the overall system reliability, the following sensitivity tests were conducted:

(a) the maximum daily peak load was varied from 60 % to 100% of the total (conventional and WECS) installed capacity,

(b) three different penetration levels for WECS at a fixed FOR = 0.05 were considered. Penetration is defined as the WECS installed capacity divided by the total (conventional and WECS) installed capacity,

(c) LOLE curves were plotted for January '88, July '88 and May '88,

(d) the impact of varying (i) the penetration level and (ii) the mean wind speed on the effective capacity [14] is made clear with reference to **Figure 7**.

At an assumed design level of 0.0083 days/month, the horizontal distance between the LOLE curve for the system without WECS and the curve for the system with WECS is the amount of load growth the system can accept and still retain the same reliability. This additional load is defined as the effective capacity, normally expressed as a percentage of the rated capacity.

Figure 8 shows that when the peak load is lower than the conventional installed capacity the influence of the WECS on the system's LOLE is not significant. As the peak load approaches the conventional generation capacity 1052 MW the LOLE increases sharply, indicating a low confidence in wind generation . The impact of penetration reduction in the system's LOLE remains constant for the 620 MW curve and decreases marginally for the 660 and 700 MW curves as the penetration increases as shown in **Figure 9**. The importance of siting the WECS at a location where the mean wind speed is a maximum is revealed by **Figures 10 and 11.**

Doubling the mean wind speed (figure 10) results in a 2% reduction in the systems LOLE at a penetration of 0.47 % and load of 650 MW. Increasing the mean wind speed 3.25 times (**Figure 11**) results in a 6% reduction in the system's LOLE under the same conditions. Hence the percentage reduction at a site where the mean wind speed is 3.25 times greater is more than 1.6 times the percentage reduction at a site where the mean wind speed is only two times greater.

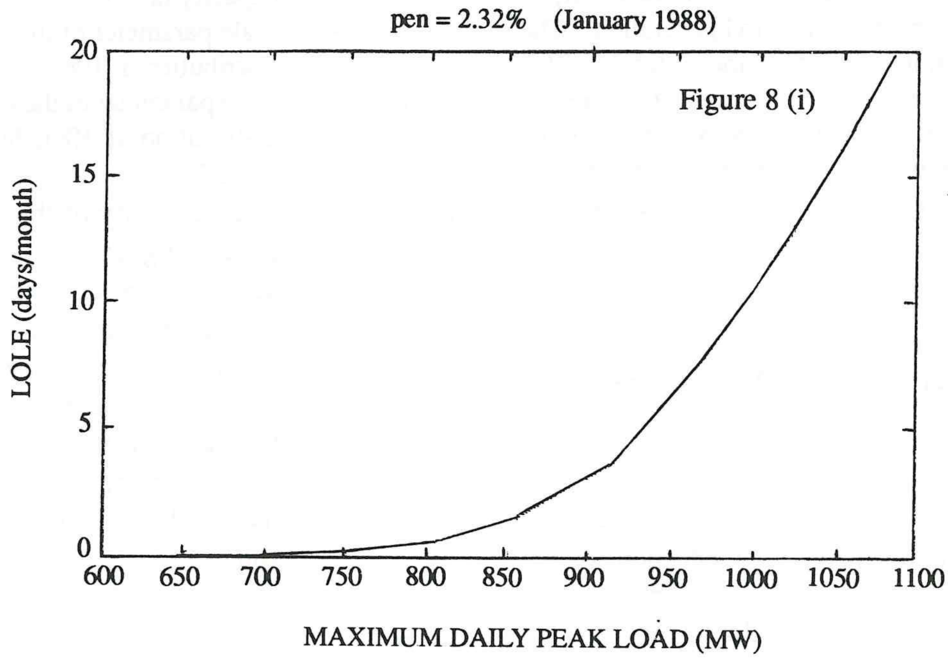
The plot of **Figure 12** is similar to **Figure 7**, but exaggerated to facilitate the estimation of the effective capacities at a varying penetration levels. The effective capacities are 51.6%, 64.6% and 63.2% for penetration levels of 0.47%. 1.41% and 2.32% respectively. This suggests an optimum effective capacity at a specific value of penetration, the effective capacity thereafter decreasing with increasing penetration levels.

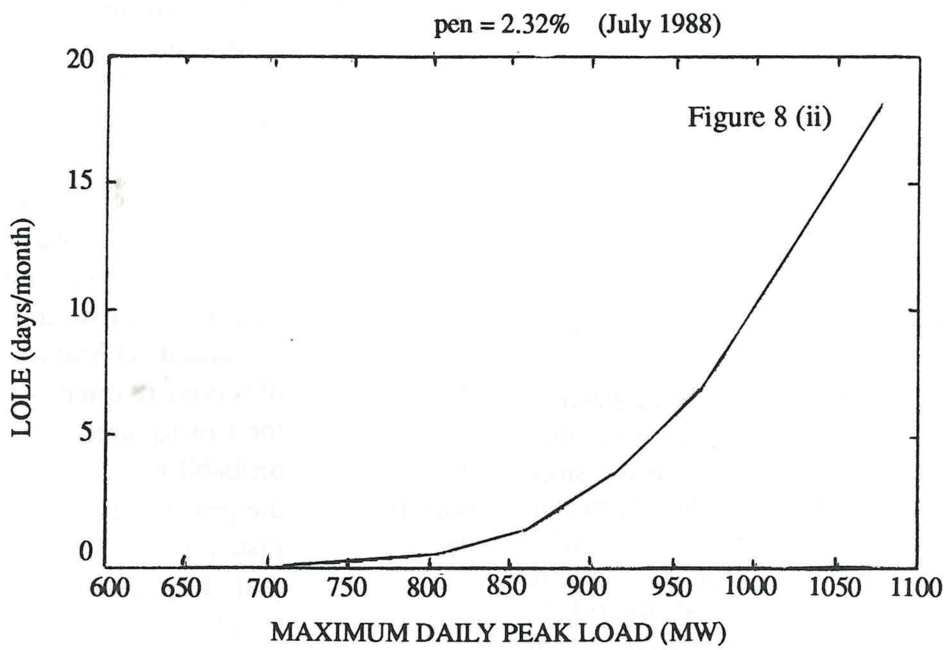
The plot of **Figure 13** also facilitates the assessment of the effective capacities for the sample months considered. The values are 3.2%, 20.8% for January and July respectively. Thus an increase in the mean wind speed by factors of 2 and 3.25 is translated to an increase in the effective capacities by factors of 6 and 20 respectively. This again illustrates the importance of siting the WECS at a location where the wind speeds are high.

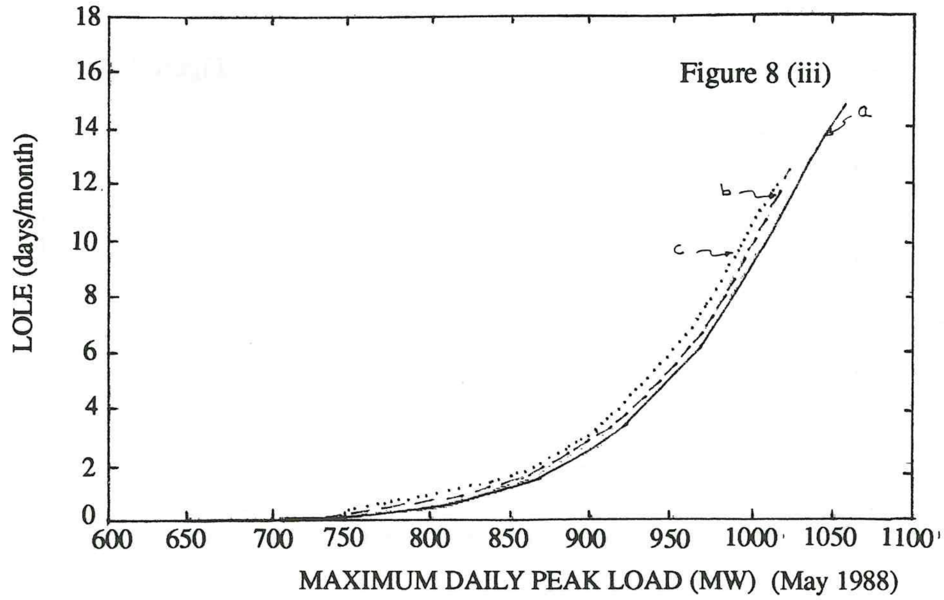
It can be concluded therefore, that the incentive to connect additional WECS to an electric utility decreases as the penetration of WECS increases since its has been discovered that both the rate of reduction in the system's LOLE and the effecitve capacity decreases with increased penetration. In addition, better wind regimes (higher mean wind speed) have been shown to enhance the reliability performance of the overall generating system.

7.0 NOMENCLATURE

c	-	scale parameter of the Weibull distribution at 10m
C	-	significance level for the Chi-squared test
CCOPT	-	cumulative capacity outage probability table
CF	-	capacity factor
cg	-	scale parameter of the Gamma distribution at 10m
$C_{hi}C_{h2}$	-	scale parameter of the hybrid Weibull distribution at 10m; hubheight of a WECS
cj	-	rated capacity of the jth generating unit
cl	-	scale parameter of the log-normal distribution at 10m
e_i	-	theoretical frequency of class i under the assumed model for the Chi-squared test
f(u)	-	probability density function
$f_h(u)$	-	hybrid Weibull probability density function
F	-	cumulative distribution function
$F_{ai}F_b$	-	cumulative probability of a capacity outage after; before a generating unit is added
FOR	-	forced outage rate
k	-	shape parameter of the Weibull distribution at 10m
kg	-	shape parameter of the Gamma distribution at 10m
$k_{hi}k_{h2}$	-	shape parameter of the hybrid Weibull distribution at 10m; hubheight of a WECS
kl	-	shape parameter of the log-normal distribution at 10m
LOLE	-	loss of load expectation
NG	-	number of Growian II turbines "installed" at Matura
oi	-	observed frequency of class i for the Chi-squared test
p	-	probability
p(u=0)	-	the probability of calms
Pe(u)	-	instantaneous electrical power output by a WECS
Pen	-	penetration level
P_{eR}	-	WECS rated electrical power output
Pe	-	WECS average electrical power output
Pn	-	normalized power

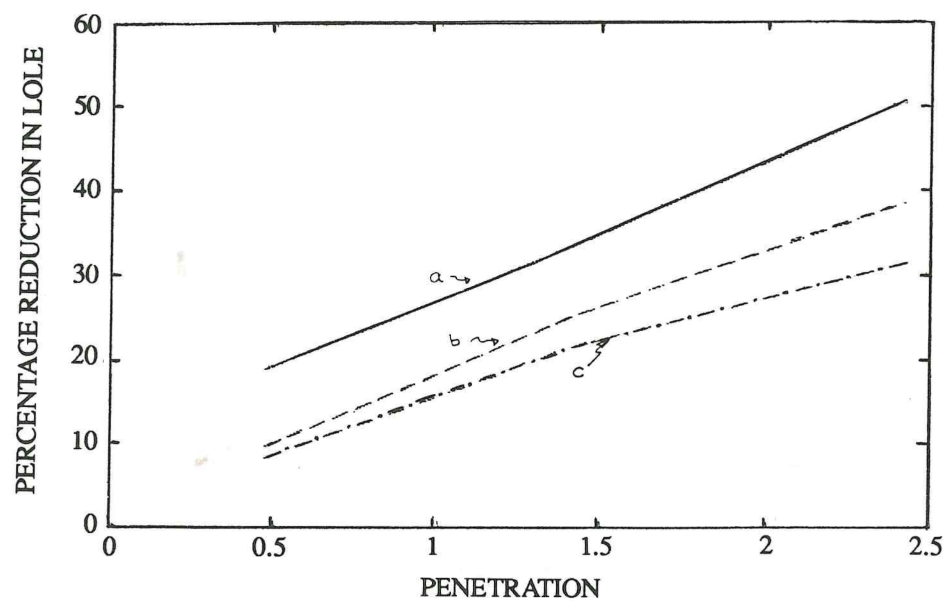

pen = 2.32% (January 1988)
Figure 8 (i)
LOLE (days/month)
20
15
10
5
0
600 650 700 750 800 850 900 950 1000 1050 1100
MAXIMUM DAILY PEAK LOAD (MW)


pen = 2.32% (July 1988)
Figure 8 (ii)
LOLE (days/month)
20
15
10
5
0
600 650 700 750 800 850 900 950 1000 1050 1100
MAXIMUM DAILY PEAK LOAD (MW)



(i) January '88
——— pen = 2.32%
(ii) July '88
——— pen = 2.32%
(iii) May '88
c ··· pen = 0.47%
b – – pen = 1.41%
a ——— Pen = 2.32%

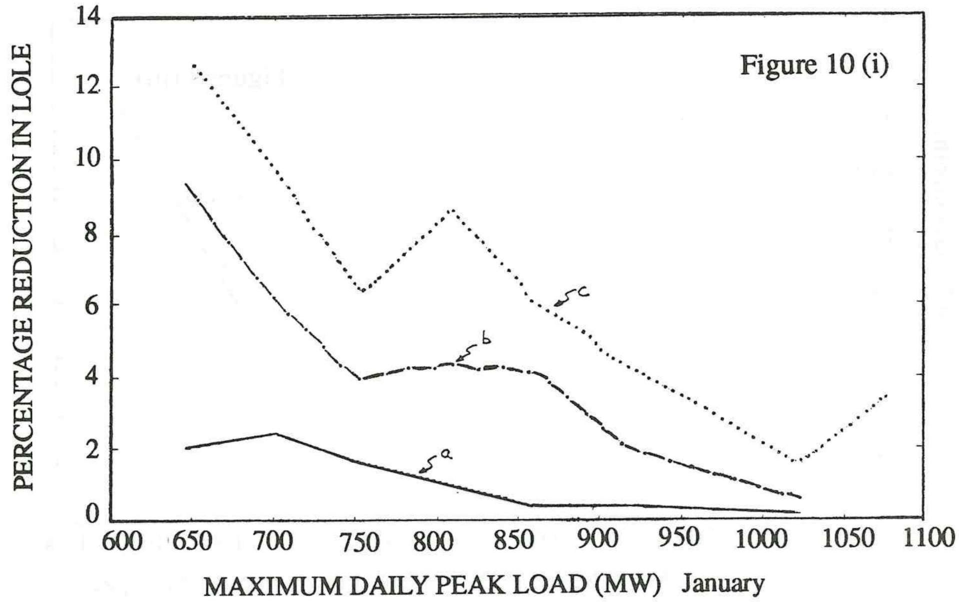
Figures 8 (i), (ii) & (iii): LOLE vs. Max. Daily Peak Load.



Percentage reduction in LOLE vs penetration, May '88

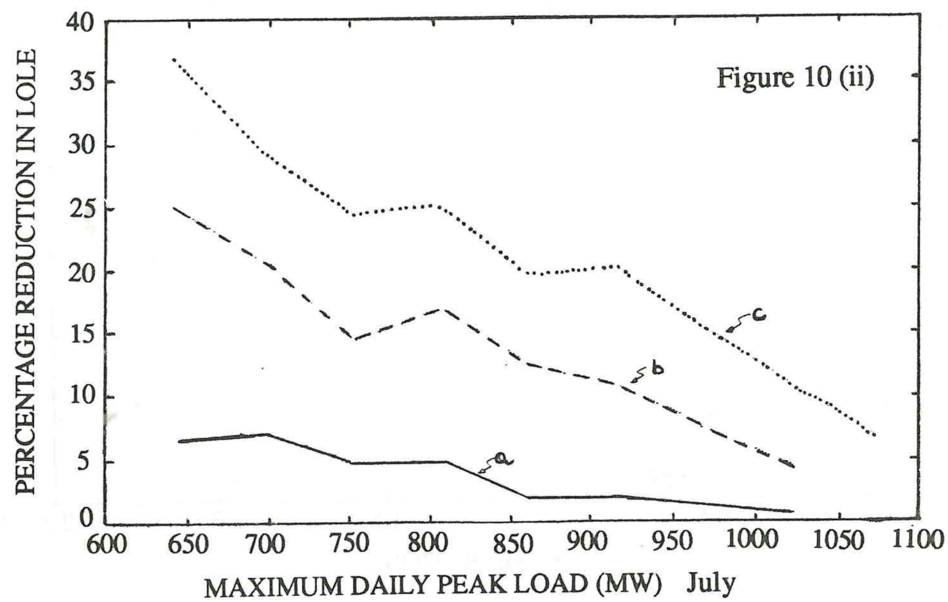
a ——— Maximum daily peak load = 620 MW
b – – – Maximum daily peak load = 660 MW
c – · – Maximum daily peak load = 700 MW

Figure 9: Percentage Reduction in LOLE vs. Penetration.



Percentage reduction in LOLE vs maximum daily peak load, January compared with July.

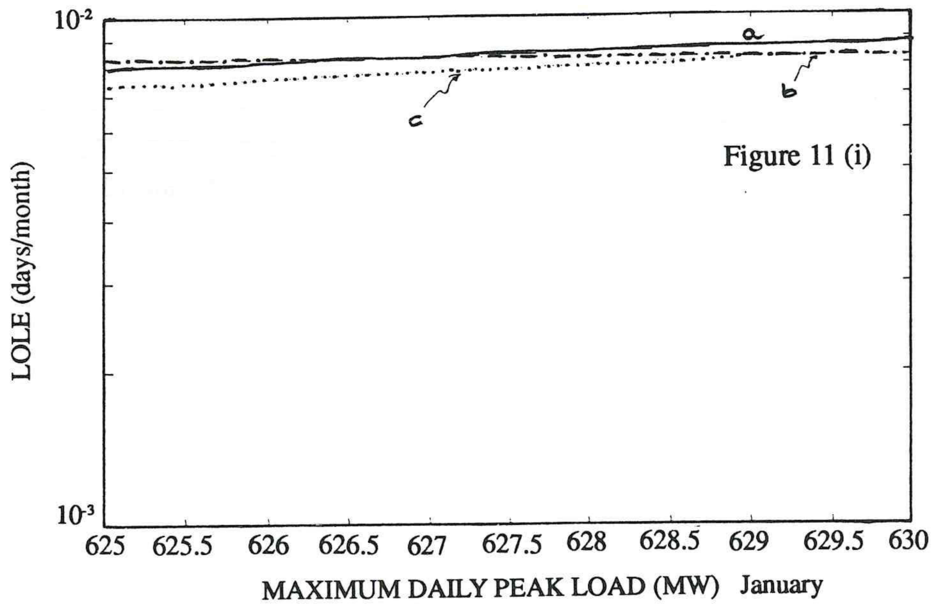
a —— pen = 0.47%
b —·— pen = 1.41%
c ·· --- pen = 2.32%

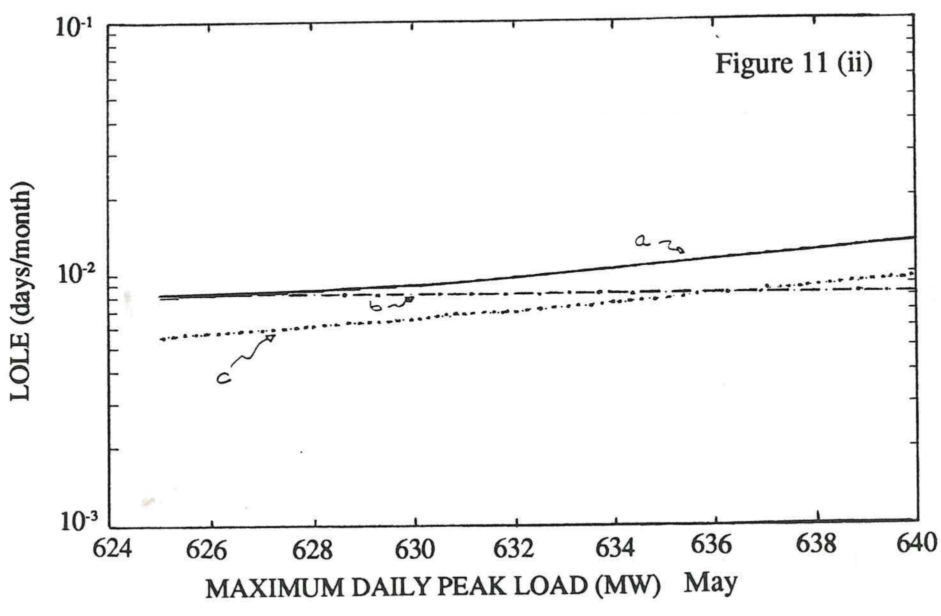


Percentage reduction in LOLE vs. maximum daily peak load, January compared with July

pen = 0.47%
pen = 1.41%
pen = 2.32%

Figures 10 (i) & (ii): Percentage Reduction in LOLE vs. Max. Daily Peak Load - January vs. July.

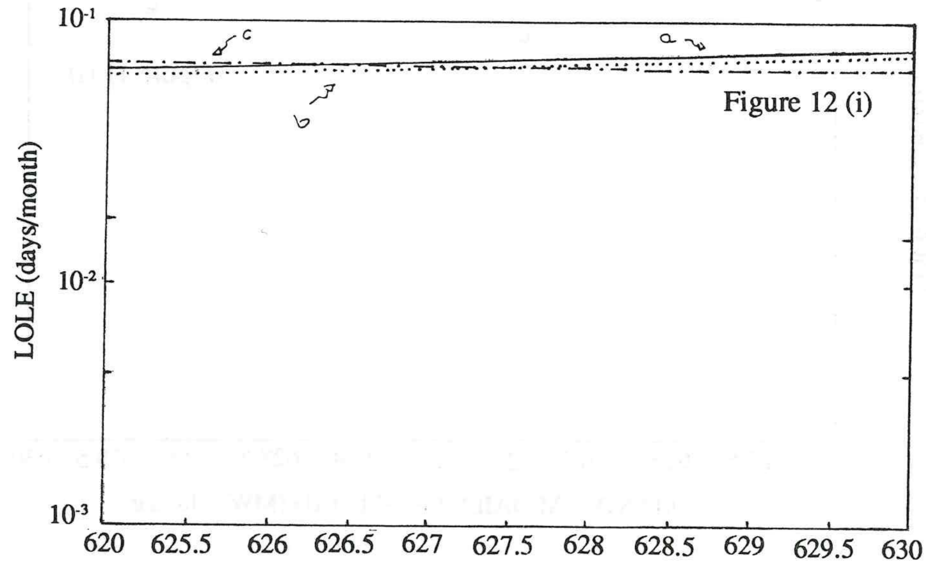




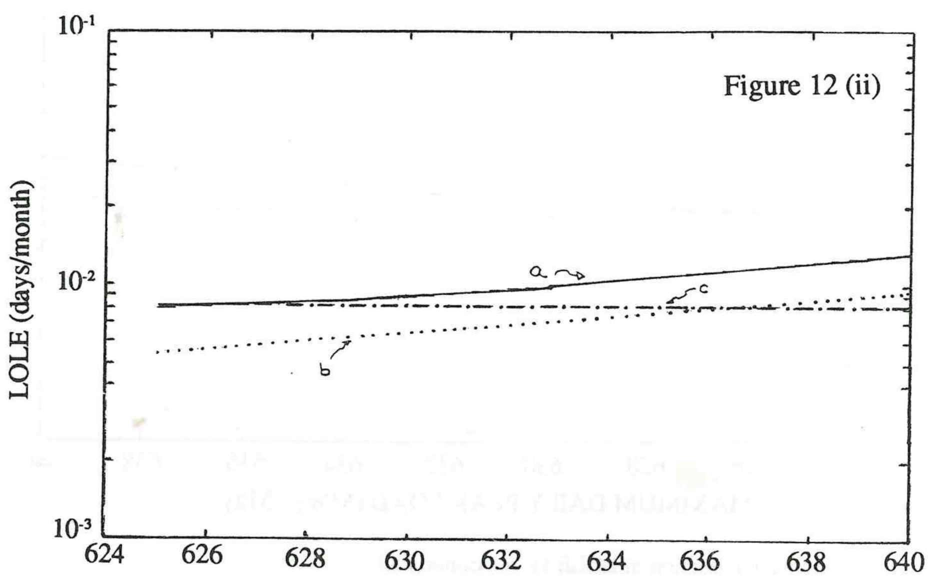
Percentage reduction in LOLE vs. maximum daily peak load, January compared with May

a —— pen = 0.47%
b —.— pen = 1.41%
c ...... pen = 2.32%

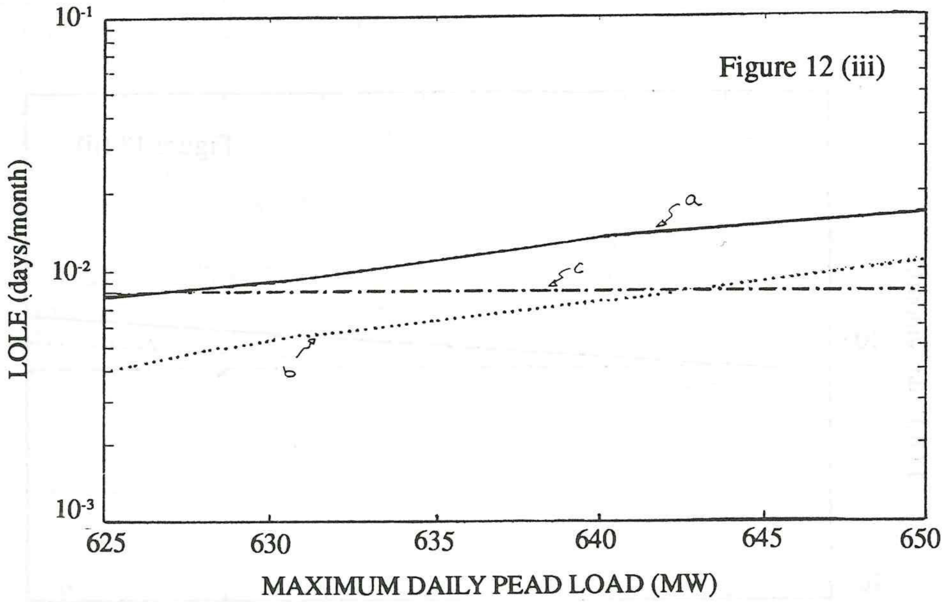
Figures 11 (i) & (ii): Percentage Reduction in LOLE for T&TEC System without and with WECS.



10-1
10-2
10-3
LOLE (days/month)
Figure 12 (i)
a
b
c
620
625.5
626
626.5
627
627.5
628
628.5
629
629.5
630
MAXIMUM DAILY PEAK LOAD (MW)



10-1
10-2
10-3
LOLE (days/month)
Figure 12 (ii)
a
b
c
624
626
628
630
632
634
636
638
640
MAXIMUM DAILY PEAK LOAD (MW)

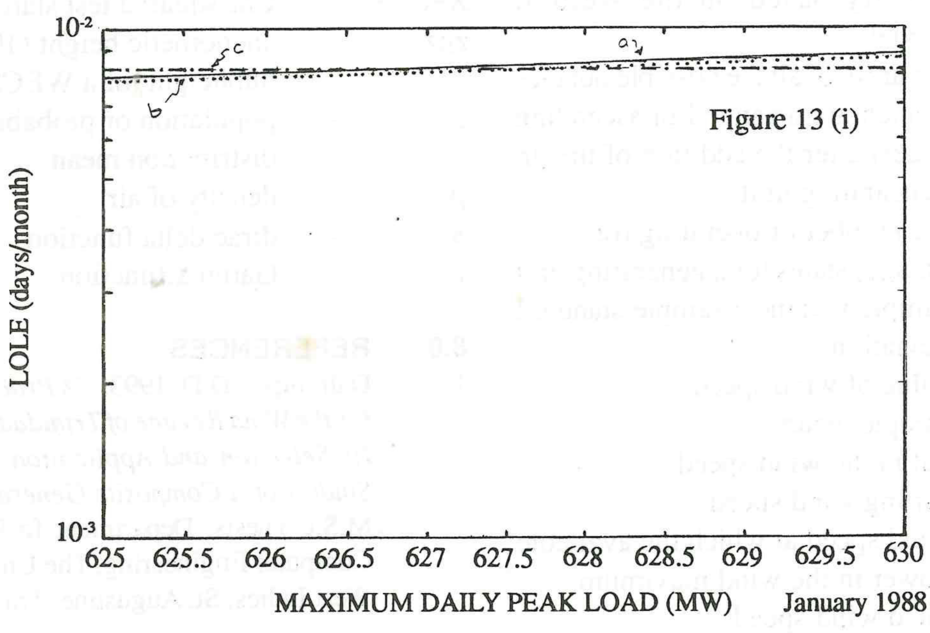


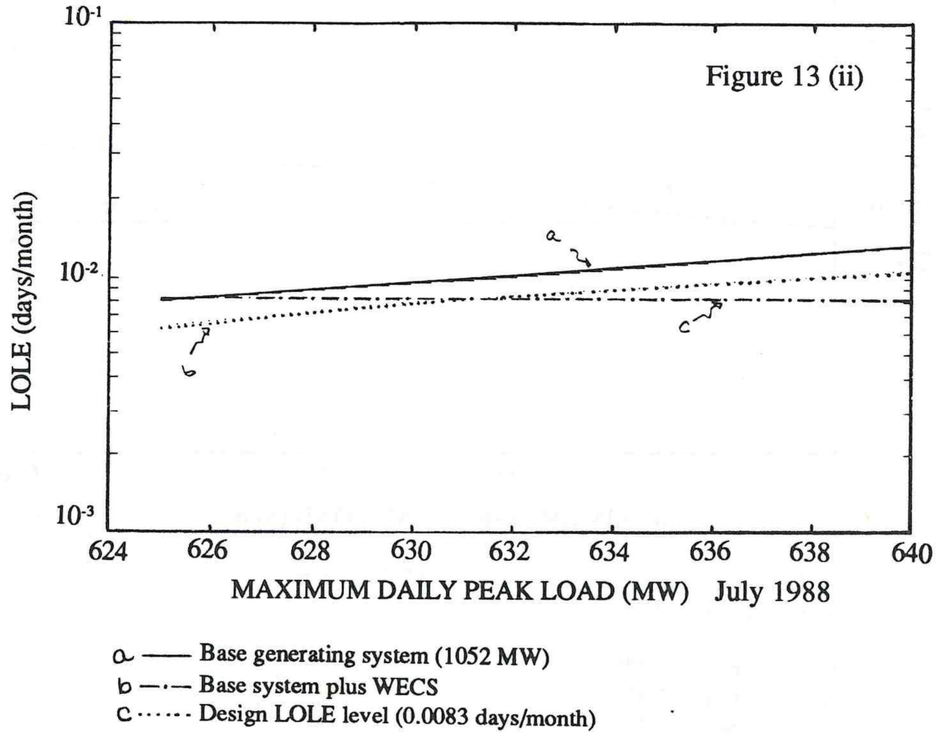
LOLE vs maximum daily peak load for T&TEC system
without and with WECS, May 1988

(i)	pen = 0.47%
(ii)	pen = 1.41%
(iii)	pen = 2.32%

a —— Base generating system (1052 MW)
b ······ Base generating system plus WECS
c —·— Design LOLE level (0.0.083 days/month)

Figures 12 (i), (ii) & (iii): LOLE vs. Max. Daily Peak Load
for T&TECSystem with/without WECS.





Figures 13 (i) & (ii): LOLE vs. Max. Daily Peak Load for T&TEC System with/without WECS (Jan. & July 1988).

Pwa	-	actual average wind power density
Pwt	-	theoritical average wind power density based on the Weibull model
PX_l	-	the array of all the possible outages (which are arranged in ascending order) after the addition of the jth genearating unit
q	-	the number of operating (or outage) states for a generating unit
s^2;s	-	sample variance; sample standard deviation
u	-	value of wind speed
u	-	sample mean
u_c	-	cut in the wind speed
u_f	-	furling wind speed
u_{mp}	-	wind speed at which the available power in the wind maximum
U_R	-	rated wind speed
w	-	the number of different values of wind speed observed
WECS	-	wind electric conversion system(s)
x_i	-	outage capacity of generating unit
X^2_c	-	Chi-squared test statistic
z_i;z	-	anenometic height (10m); hubheight of a WECS
μ	-	population or probability distribution mean
ρ	-	density of air
δ	-	dirac delta function
Γ	-	Gamma function

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