USE OF FATIGUE DATA IN DESIGN

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Summary

The strength properties of constructional materials are measured by tests in which simple stresses are applied. This information has to be used for proportioning machine parts that have to resist failure under working loads. A typical part works under a stress cycle consisting of fluctuating and constant components. Rules for applying test data in a design problem should be consistent with the physics of the failure process. Fatigue failure in metals is caused primarily by the range of applied shear stress. This is discussed here, together with the effect of other stress components.

1. INTRODUCTION

It is often claimed that eighty to ninety percent of service failures of machine parts can be attributed to fatigue [1]. Although this topic has been studied intensively over sixty years or more, there does not appear to be any universal agreement about design procedure. Many texts follow an intuitive idea that breakage is caused by tensile stress, even though the history of metal fatigue emphasises the importance of shear stress.

The crystalline structure of metal orginates in cooling from the molten state and persists through mechanical working processes. The orderly arrangement of atoms within a crystal can be revealed by etching a polished sample that ıas been lightly cold-worked. The slip bands that are seen are straight and parallel over the whole width of a crystal or ɔver part of the width. Unidirectional slip is not necessarily damaging, but when reversed slip occurs repeatedly, coıesion between the walls of a slip band is lost and a crack is formed. While this oversimplifies all the detailed work that has ɔeen done on the initiation of fatigue cracks, it directs attention to the significance of relative movement between the valls of a slip surface.

2. RELATIVE MOVEMENT IN A SLIP BAND

Figure 1(a) shows shear stress s acting on the edge of an infinite plate containing a stress-free crack of length 2a. 'he maximum horizontal movement of the upper crack surface occurs at mid-point P, this being u = 2a s/E. This tandard result of elastic stress analysis is most easily confirmed by the use of a suitable stress function of the Vestergaard type. This expression is for plane stress, and if the value for plane strain is required, a modified elastic onstant E should be used. Figure 1(b) shows a state of uniform stress in which a positive shear stress k acts on the xterior surfaces of the plate also on the surface of the crack. Here, the displacement of point P is zero. Corresponding :ress components and displacements for these two situations are now added. The result gives the relative movement between point P on the upper surface of the slip plane and the point directly opposite on the lower surface when a ıear stress q is applied at the outside edges and a stress k acts on the slip surface tending to oppose relative moveıent,

$$U = 2u = 4a(q - k)/E, \quad (q > k) \qquad \text{(1)}$$

Shear stress k may be thought of as a characteristic value for the material which is just sufficient to cause local p in a slip band that is aligned with the plane of maximum applied shear stress. It is not necessarily the same as the ear stress required to cause gross yielding in polycrystalline metal; but is expected to increase with increasing hard:ss. Equation (1) suggests the existence of a threshold value of applied shear stress q below which the amplitude of p displacement is either zero or is insufficient to cause cumulative damage. Hence the experimental curve of alter.ting stress versus number of cycles to cause failure is expected to show a definite fatigue limit. For metals, this is und to be more or less correct, subject to metallurgical changes that may be induced by cyclic plastic deformation.

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For example, most steels show positive hardening due to strain-ageing, leading to a sharp fatigue limit, whereas alu nium alloys tend to soften due to over-ageing and show no definite fatigue limit. In passing, it is noted that Equa (1) suggests a more rapid increase of cumulative slip in proportion to the length 2a of the active slip band.

3. EFFECT OF A SEQUENCE OF SHEAR STRESSES

The model of slip band displacement suggested by Equation (1) is now used for predicting cumulative slip at centre of a slip band enclosed within the stressed body, but a similar result will apply to a half-band terminating o surface of the stressed body. Applied stress q is taken to consist of a cyclic mean value q_m plus an amplitude variation q_a. Figure 2 shows the applied stress varying with time in the form of a series of straight lines, but the ex way in which it varies is immaterial. The corresponding amounts of displacement of the slip surfaces are represented a scale such that they are proportional to stress differences.

On loading the initially stress-free piece, no slip will occur until the shear stress on the slip plane reaches value k, point A. Slip then continues up to point B. On unloading the elastic stress distribution has to change by an amount before the stress acting on the slip plane becomes reversed in sign, at point C. Reverse slip then occurs, by an amou proportional to $2(q_a - k)$ until point D is reached. Subsequently, this same amount of slip will occur during each ha cycle.

Damage arises only from reversed slip repeated many times. The argument presented here predicts that damage wi depend only on the range of applied shear stress and not on its mean value. This is well confirmed experimentally, as i the results of J.O. Smith reproduced by Forrest [2].

4. EFFECT OF MEAN TENSILE STRESS

If tensile stress σ_n is applied in a direction normal to the slip plane during the time when slip is proceeding, it has the effect of lowering the value of the shear stress amplitude q_f required to cause failure by fatigue cracking. If the effect is relatively small, it can be taken to be a linear effect, expressed by

$$q_f = q_o - \alpha \sigma_n \qquad (2)$$

where q_o is a basic value and α is a numerical constant. A formulation os this kind was used by Findlay [3].

Many quasi-static tests have been carried out on torsion specimens subjected to hydraulic pressure. Generally, the flow curve has been found to be unchanged, but an increasing all-round pressure increa ses the strain at which fracture occurs. It seems likely that the presence of a tensile stress acting prependicularly to the shear plane in a fatigue test similarly has no effect on the shear stress required to cause reversed slip, but reduces the cumulative slip required for the formation of a crack. Consider, for example, a non-ferrous specimen being tested at a stress expected to cause failure in 10^7 cycles. Increased tensile stress normal to the shear plane will presumably leave the amount of slip per cycle unchanged, but will increase the rate at which the slip causes permanent damage. The effect will be to lower the curve of stress versus number of cycles to failure.

As an example of representing fatigue data in the form suggested by Equation 2, results for mild steel specimens given by Frost, Marsh and Pook [4] are plotted in Figure 3. These results appear to fitted by the expression

$$\sigma_a = 190 - 0.15\sigma_m \quad (N/mm^2) \qquad (3)$$

where σ_a is the amplitude of tensile-compressive stress at the fatigue limit and σ_m is the mean tensile stress of the cycle. For this uniaxial stress cycle, $q_f = \frac{1}{2}\sigma_a$ and $\sigma_n = \frac{1}{2}(\sigma_a + \sigma_m)$. Comparison of Equations (2) and (3) gives $\alpha =$ 0.18 and $q_o = 112$ N/mm².

In Figure 3, a line drawn at an inclination of 45° shows points where the maximum tensile stress of the cycle reaches the tensile strength (410 N/mm²). At these points the specimen fails due to progressive stretching in the manner of a specimen under quasi-static loading, without the formation of a fatigue crack. Therefore, this condition limits the range of valid fatigue strength data obtainable from this kind of test.

From the above treatment of tensile fatigue test data, it follows that the ratio of the fatigue limit in reversed orsion ($\sigma_n = 0$) to that in reversed tension is $\frac{1}{2}(1 + \alpha)$. From the above results for mild steel, this ratio has a pre-icted value 0.59. Evidence in the literature of fatigue testing shows values of about 0.60 for wrought steels, down to bout 0.55 for wrought non-ferrous alloys. It is sometimes suggested that these experimental values support the use of he von Mises criterion of failure, which predicts a ratio 0.577. This is not in keeping with the ideas adopted here, for he reason that superposition of an all-round stress leaves the von Mises criterion unchanged, but would be expected o change the fatigue strength.

SELECTION OF SHEAR PLANE

The idea of a shear stress amplitude τ_e which is effective in causing fatigue cracking can be formulated by trans-osing the terms in Equation (2),

$$\tau_e = \tau_a + \alpha\sigma_n$$

where τ_a and σ_n are the calculated values shear stress amplitude and normal tensile stress acting on some selected plane. The plane on which the effective stress τ_e is a maximum will not be precisely the same plane on which the shear stress mplitude τ_a is a maximum. It is suggested that the more complicated calculations required to determine this plane of ninimum strength are not necessary for the following reasons. The shear stress amplitude has a stationary value on planes rotated away from the plane on which it is a maximum, so a small rotation will change its value by only a negligible amount. The tensile stress normal to the plane of maximum shear stress amplitude does not have a stationary value with respect to small rotations, but the small change in its value due to a small rotation is multiplied by the small constant α, giving a change of the second order of smallness. Therefore the rotation of the reference plane from the position of the plane of maximum τ_a to the plane of maximum τ_e is likely to lead to only a negligible change in the value of τ_e. It will be a useful simplification of calculations to select the plane of maximum shear stress amplitude, and to use stresses referred to this plane

6. THE DESIGN PROBLEM

In the usual engineering situation, the working loads lead to values of stress components at a selected point on the surface of a load-carrying element. This surface usually has no normal or shear stress externally applied to it, so the stress system is biaxial, consisting of two direct stresses and a shear stress referred to specified co-ordinate axes. By the use of a stress circle, the magnitude and direction of the principal stresses can be found.

It should be remembered that if the principal stresses are of opposite sign, the planes of maximum shear stress will be at 45^o to the principal axes and at right angles to the surface. This is so in shaft problems where the transverse stress is zero. In this case, the maximum shear stress is half the algebraic difference between the principal stresses. However, if the principal stresses are of the same sign, the planes of maximum shear stress will be at 45^o to the axis of the greater principal stress and also to the surface. In this case, the maximum shear stress is half the value of the greater principal stress. When the plane of maximum shear stress has been established, the tensile stress acting normal to this plane may be found. In the first of the above situations, this will be equal to the average of the principal stresses. In the second case, it will be equal to half the value of the greater principal stress. When the alternating components of stress have been dealt with in this way, it may be necessary to draw a new stress circle for the static or mean stresses. The dynamic and static values of tensile stress normal to the shear plane are then added to obtain the cyclic peak value σ_n.

When the shear stress amplitude τ_a and the maximum normal stress σ_n have been found by calculation, the effective shear stress amplitude τ_e can be found from Equation (4). The outcome of a stress analysis is usually a safety factor, which compares the stress due to working loads with the strength of the material used. The safety factor is the factor by which all the calculated components of working stress have to be scaled in order to reach a fatigue failure condition. This factor emerges from the above calculation in a particular simple way, as it is given by q_o / τ_e, where q_o is the basic shear stress amplitude for the material, derived from test results according to Equation (2).

7. CONCLUSIONS

1. An expression is derived for quantifying the amount of damage sustained by a small slip band enclosed within an otherwise elastic body.
2. While fatigue damage is primarily decided by the maximum amplitude of shear stress, it is accelerated by the

presence of a tensile stress acting normal to the shear plane.

3. A design procedure is suggested using the calculated orientation of the plane of maximum shear stress amplitud
4. Calculated stresses and normalised fatigue test data lead directly to a safety factor.

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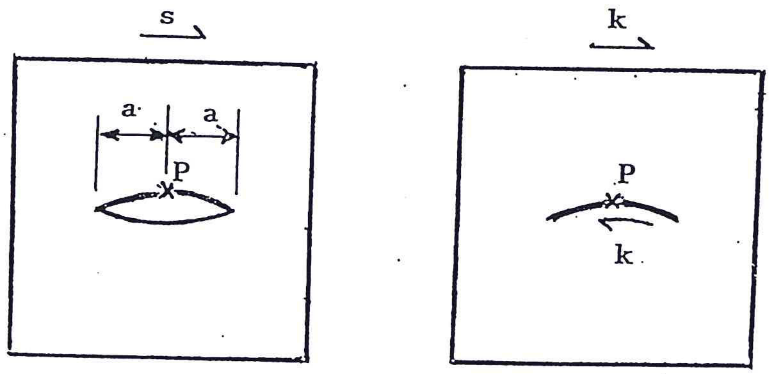

(a) (b)

Figure 1

(a) Crack in plate under shear stress,

(b) Uniform shear stress

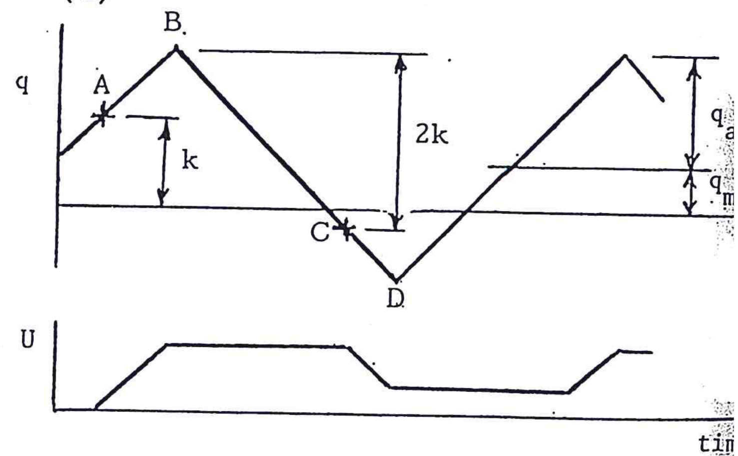

Figure 2

Slip displacement U caused by applied shear stress q

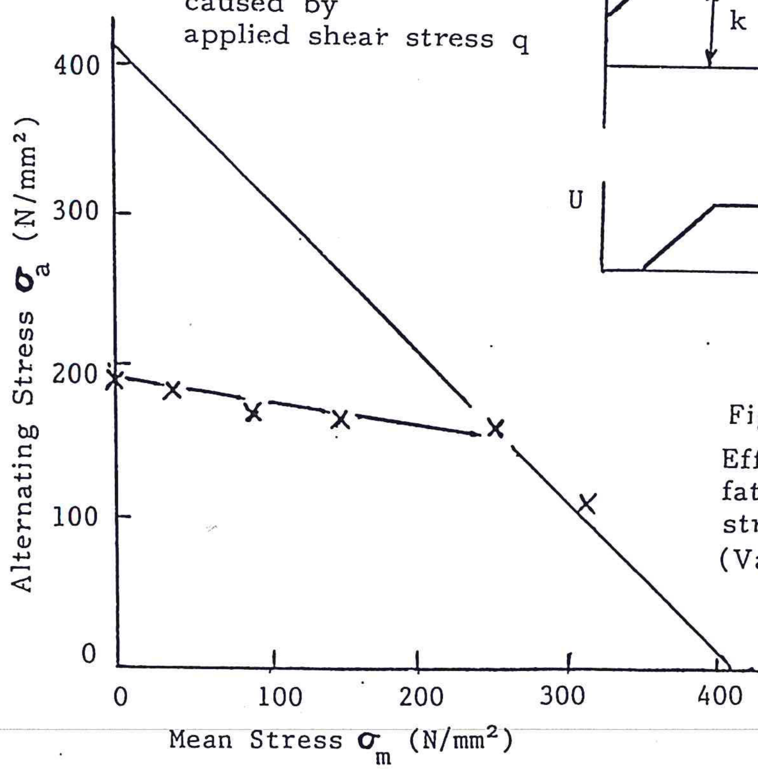

Figure 3

Effect of mean stress on the fatigue limit value of alternating stress amplitude. For mild steel. (Values from Ref.4)