A NONLINEAR ANALYSIS OF METAL CUTTING IN THE BUILT-UP EDGE REGION

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Summary

In this paper a nonlinear model for metal cutting in the built-up edge region has been proposed. This nonlinear model is investigated with the help of an analog computer. The results of the analog simulation have been presented in this paper.

1. INTRODUCTION

When cutting mild steel, aluminium and other ductile materials under certain conditions built-up edge (BUE) often appears on the tool. Under these conditions the friction between the chip and the tool is so great that the chip material welds itself to the tool face. The resulting pile of material is known as BUE. BUE is generally a more or less fixed wedge shaped body of material located at the cutting edge. The cutting speed has a great influence on the formation of BUE. Figure (1) shows the typical relationship between the height of the BUE and the cutting speed [1,2,9,10]. In the low speed range O-V_1 there is no BUE. This speed range is used in the case of reaming and broaching operations. In the speed range from V_1 to V_2, the stable BUE is noticed, while in the speed range V_2-V_3, the BUE periodically breaks off giving rise to a variable cutting force. This particular region is known as unstable BUE. Under this condition self induced vibrations or chatter occurs. When the cutting speed is more than V_3, there is no more formation of BUE. For finishing operations it is always recommended to use higher cutting speed (more than V_3). Figure (2) shows the relationship between the height h of the BUE and the underformed chip thickness in turning steel 45 at different speeds [2]. Steinberg [3,4] and Kuchma [6] obtained interesting results on the relation between BUE and vibrations while turning steel 45. The stability of BUE is a particular case of the stability of plastic deformation in cutting and is determined by the parameters of that process. When the BUE is unstable the cutting force varies which ultimately gives rise to chatter. Unstable BUE is undesirable because of its adverse effects on surface finish, machining accuracy and tool life. Investigations into machining with unstable BUE is important, because materials are often machined with unstable BUE. Analysis of chatter due to unstable BUE requires the solution of a nonlinear model. Mathematical models in the form of transfer function describing the dynamic cutting process with emphasis on orthogonal cutting have been proposed by many researchers [5,7]. However, most of these models are valid only in the absence of BUE i.e. in the steady state of cutting (linear model). In the present paper a nonlinear model for metal cutting with BUE has been proposed. This nonlinear model is investigated with the help of an analog computer.

The analog computer is an invaluable tool for the analysis of a nonlinear system. Most of the analytical and graphical techniques that are available for the analysis of nonlinear systems give the engineer a limited amount of information about a system's behaviour. In other words they can be used to establish the nature of system stability, and they can, in general, give some information about the system response for certain types of simple inputs.

The analog computation on the other hand is flexible and effective tool for investigating a nonlinear system. The reason is that the typical nonlinear functions are easily generated with an analog equipment.

The nonlinear model which is proposed is investigated with analog simulation. The chatter behaviour of the machine tool has been investigated through simulation.

2. MODEL PROPOSED

Figure (3) shows the simplest model of dynamic systems of a machine and its interrelation of the elestic system with the cutting process. The model shows that the relative vibration between the tool and the workpiece changes the cutting process so as to cause a variation in cutting force P acting on the equivalent elastic system (EES) and again creates vibration y. However, this simple model is not enough to explain self excited vibration or chatter in machine tools due to cutting operation. Analysis of chatter due to metal cutting requires an analysis of a nonlinear model. Figure (4) shows the BUE characteristic in turning steel. Formation of BUE changes the tool angles (increases rake angle). Hence the magnitude of the cutting force, chip contraction factor changes due to BUE. Since the BUE occurs between the chip and the tool face it changes the direction of the movement of the chip. By the time it leaves the chip formation zone, the chip is no longer moving parallel to the tool face, but approximately parallel to the mean face of the BUE. Actual rake angle is determined by the angle between the cutting speed vector and the chip speed vector. This change in direction of the chip velocity leads to an increase in the actual rake angle. A peculiarity of the BUE is that it can not increase the actual rake angle to more than 45°. Therefore, the influence of BUE on the actual rake angle and the cutting process is reduced at large rake angles and increased at small rake angles.

The BUE formed by strongly deformed and workhardened work material, displaces the line indicating chip separation from the parent metal into the workpiece, that is, the BUE acts as a cutting tool and the part a_b of the BUE extending beyond the tip of the tool increases a_o, the thickness of the undeformed chip. [See Figure 4]. The change in the undeformed chip thickness caused by the projecting part a_b in turn produces change in chip thickness a_1. The transfer function of the chip formation which is the ratio of the output parameter (in this case change in the chip thickness a_1) to the input parameter (change in the thickness of the undeformed chip a_b) may be expressed as:

$$W_c = \frac{a_1}{a_b} = \frac{\xi_o}{T_c p+1} \qquad \ldots\ldots (1)$$

As the chip thickness increases the length of contact between the chip and the tool rake face will also increase giving rise to more BUE formation (h_c) which is reflected in the model (Figure 5). The transfer function of BUE formation W_h due to increasing chip thickness may be expressed similarly as:

$$W_h = \frac{h_c}{a_1} = \frac{\xi_h}{T_h p+1} \qquad \ldots\ldots (2)$$

The change of chip thickness a_1 will also change the cutting force as shown in the model (Figure 5) and this transfer function is equal to:

$$\frac{P}{a_1} = \sigma_o \cdot b \qquad \ldots\ldots (3)$$

The variation in the cutting force P will act on the EES changing the relative position of the tool and the workpiece (y). The transfer function of EES may be expressed as

$$W_{EES} = \frac{P}{y} = \frac{K_{EES}}{T_1^2 p^2 + T_2 p + 1} \qquad \ldots\ldots (4)$$

(To simplify the analysis the characteristics of one of the normal modes of vibration with its natural frequency is used).

The increase in y corresponds to a reduction in undeformed chip thickness a_o. Analysing the relationship between the height h of the BUE and undeformed chip thickness a_o, it may be concluded that change in y due to variation of cutting force will in turn change the height hy of the BUE (see Figure 5) and its transfer function is given by:

$$K_b = \frac{h_y}{y} \qquad \ldots\ldots (5)$$

The BUE height, h_c and h_y are changed due to two causes as explained above. Both of them are algebraically added up in the model to have the total change in the height of BUE (h_a), which in turn affects the variation in dimension a_b.

In the unstable BUE the periodical rupture of the BUE gives rise to a nonlinearity [8]. Stages I-IV of this process and the relationship between a_b and h_a are shown in Figure 6. The same Figure shows the simplified non-linear characteristic of the BUE formation process, which is obtained by replacing the continuous curve (broken lines) by a straight line section (continuous lines).

The nonlinear model has been analysed theoretically using the procedure of harmonic linearization [5,7]. In the present paper the model is investigated with the help of an analog computer.

3. ANALOG SIMULATION OF THE NONLINEAR MODEL

The schematic of the nonlinear model investigated is shown in Figure (5). The model is analysed with the help of an analog computer. The block diagram is shown in Figure (7). The nonlinear part of the model, the formation and rupture of BUE is achieved with the operation relay ORB (see Figure 7). The investigation is carried out for the following values of major dynamic parameters.

$$C_{EES} = 4000\ N/mm, \quad f_n = 80\ Hz, \quad \lambda = 0.6, \quad \xi_h = 2,$$

$$\xi_o = 4, \quad T_1 = 2 \cdot 10^{-3} sec, \quad T_2 = 0.4 \cdot 10^{-3}\ sec$$

$$T_c = 4.5 \cdot 10^{-3} sec, \quad T_h = 14 \cdot 10^{-3} sec$$

In the unstable BUE region, the BUE height decreases with an increase in the undeformed chip thickness. Therefore the transfer function K_b is negative which is reflected in the block diagram. The sign of h_c and h_y are different, because the increase in y corresponds to a reduction in the undeformed chip thickness.

The behaviour of the machine tool for the following two particular cases are investigated:

i) $\frac{K_p}{C_{EES}} > 1$ and ii) $\frac{K_p}{C_{EES}} < 1$.

The physical significance of these two cases are:

i) $\frac{K_p}{C_{EES}} > 1$, this particular case of machining may be observed while machining with high width of cut (b) and low rigidity of the elastic system of the machine tool, eg. for rough machining.

ii) $\frac{K_p}{C_{EES}} < 1$, may be achieved with small width of cut but high regidity of the elastic system eg. for finish machining.

In order to change the equations of the proposed model into machine equations the following scale factors are selected:

$$m_x = 0.1 \frac{V}{N}$$

$$m_y = 200 \frac{V}{mm}$$

$$m_t = 3.10^2$$

For simulation purposes, all the coefficients K_i in the block diagram are calculated taking into account scale factors. The values of these coefficients are as follows:

$$K_1 = \frac{K_{EES}}{T_1^2} \cdot \frac{m_y}{m_x \cdot m_t^2} \qquad K_7 = \frac{\xi_h}{T_h \cdot m_t}$$

$$K_2 = \frac{T_2^2}{T_1^2} \cdot \frac{1}{m_t} \qquad K_8 = 0.1$$

$$K_3 = \frac{1}{T_1^2} \cdot \frac{1}{m_t^2} \qquad K_{12} = \frac{\xi_o}{T_c \cdot m_t}$$

$$K_4 = K_5 = 1 \qquad K_{14} = \sigma_o \cdot b \cdot \frac{m_x}{m_y}$$

$$K_6 = 2.5 \qquad K_9 = K_{10} = K_{11} = K_{15} = K_{17} = 1$$

The following parameters are recorded on an eight channel automatic recorder connected to the output of the analog computer:

(i) frequency of vibration (f) of the system,
(ii) amplitude of vibration (y),
(iii) cutting force P.
(iv) change in the undeformed chip thickness a_b,
(v) height of the BUE h_a.

The dynamic behaviour of the machine tool under different values of $\frac{K_p}{C_{EES}}$ is investigated. The results of $\frac{y}{P}$ with $\frac{K_p}{C_{EES}}$, $\frac{h_a}{a_b}$ with $\frac{K_p}{C_{EES}}$ and f with $\frac{K_p}{C_{EES}}$ are represented in Figure 8 (a), (b), and (c).

4. DISCUSSIONS AND CONCLUSIONS

The dynamic behaviour of the machine tool system in the unstable BUE region while finish cutting is different from that roughing cuts. In case of finish cuts $\frac{K_p}{C_{EES}} < 1$. The frequency of the self excited vibration in the machine-fixture-tool-workpiece increases with the increase of the value $\frac{K_p}{C_{EES}}$. On the other hand the frequency of the self excited vibration for roughing cuts $\frac{K_p}{C_{EES}} > 1$ remain practically constant and near the value of the natural frequency of the system [see Figure 8 (c)].

The stability of a BUE is a particular case of the stability of plastic deformation in cutting. In the proposed model, the vibration of the machine tool (y) reacts with the self-excited vibration of the cutting process [periodic formation and rupture of BUE]. In nonlinear mechanics this type of problem is similar to the interaction of a nonlinear generator of vibrations (unstable BUE) and the vibrating system (elastic system). In case $\frac{K_p}{C_{EES}} < 1$, width of cut, b is small and the elastic system is of high stiffness and hence vibration of the elastic system will also be small. That is why cutting process characteristic predominates in this particular case, resulting in increase in the frequency of self excited vibration with the increase of $\frac{K_p}{C_{EES}}$.

However when $\frac{K_p}{C_{EES}} > 1$ i.e. with wide undeformed chip and the elastic system of low stiffness, the vibration of the elastic system will be considerable and hence the frequency of self excited vibration remains almost constant and near the value of the natural frequency of the system.

The results obtained explain the conclusion that the stiffness of the system and depth of cut affects the frequency at which the BUE is detached [3].

In the absence of an unstable BUE and the formation of continuous chip, self excited vibration may occur due to loss of motion stability. This has been proved when turning with cutting tool preheated up to 500°C. and thus artificially eliminating the BUE [7].

If chatter takes place during turning in the absence of BUE formation, the analysis of stability of the machine tool should be done by the model proposed in Figure 3. Deformation y in the EES of the machine directly changes the thickness of the undeformed chip and it is in this form that the system is considered in the dynamic calculations and analysis of the machine tool.

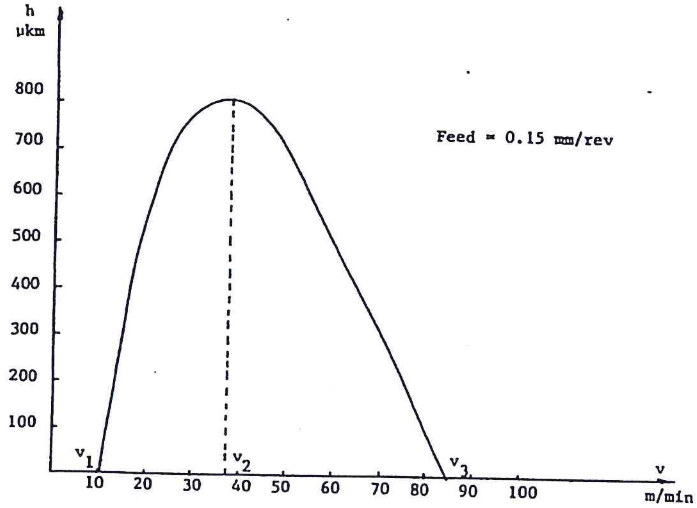
NOMENCLATURE

a_o — undeformed chip thickness
a_b — change in the undeformed chip thickness caused by the projecting part of BUE
a_{lo} — thickness of the chip
a_l — change in the chip thickness resulting from a_b
b — width of the undeformed chip
h — height of the BUE
h_a — total change in the height of the BUE
h_c — change in the height of the BUE due to change in the chip thickness a_l
h_y — change in the height of the BUE due to relative displacement of the tool and the workpiece (y)
f_n — Natural frequency of vibration of the system
y — relative displacement between the workpiece and the cutting tool (i.e. amplitude of vibration)
K — specific cutting force in N/mm^2
K_p — specific resistance to cutting = $K.b = \sigma_o \cdot \xi_o \cdot b$
K_b — ratio between the change in the height of the BUE to the change in the thickness of cut
P — variation in the cutting force
V — cutting speed in m/min
p — $\frac{d}{dt}$ operator symbol
T_l — $\frac{1}{W_y}$, Inertial time constant (inverse value of the natural frequency W_y of the system) in sec
T_2 — T_1 λ is the damping time constant in sec
T_c — $\frac{a_o \xi_o}{V}$, chip formation time constant in sec, where V is in mm/sec
T_h — BUE formation constant in sec
C_{EES} — rigidity of EES IN N/m
K_{EES} — $\frac{1}{C_{EES}}$ = factor expressing the compliance of EES
W_h — transfer function of the BUE formation
W_c — transfer function of the chip formation
W_{EES} — transfer function of the elastic system
σ_o — ultimate strength of the material in N/m^2
λ — log decrement of the system
ξ_o — chip contraction factor

ξ_n — chip contraction factor during BUE formation

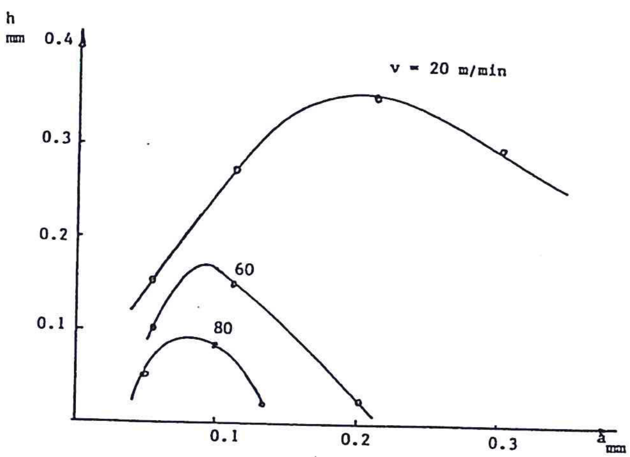
REFERENCES

1. ZOREV, N.N., "Metal cutting mechanics" Pergamon Press, 1966.
2. EREMIN, A.N., "The Physical Basis of Phenomena in cutting steel", Mashgiz, 1954.
3. KUZNETSOV, V.D., "Metal Transfer and Build-up edge in Friction" Pergamon Press, 1966.
4. SHTEINBERG, I.S., "The ellimination of chatter in lathes", Mashgiz, 1950.
5. KUDINOV, V.A., "Dynamic of Machine Tools", Machinostroenie, 1967.
6. KUCHMA, L.K., "Experimental Investigations on chatter in Lathes" Mashgiz, 1948.
7. BANDYOPADHYAY, B.P., "Investigation of Machine Tool chatter and characteristics of Dynamic cutting process in the Built-up edge region", Ph.D. Thesis, People's Friendship University, Moscow, 1979.
8. LOLADZE, T.N., "Chip Forming in Metal Cutting", Mashgiz, 1952.
9. HEGINBOTHAM, W.B. and GOGIA S.L., "Metal cutting and the Built-up edge", Proc. Instn. Mech. Engrs., Vol. 175, No. 18, 1961.
10. CHOUZE-HUA, "The effect of cutting conditions on the Built-up edge" Proc. 21st International Machine Tool Design and Research Conference, Macmillan, 1981.
11. TRENT, E.M., "metal Cutting", Butterworths, London 1977.



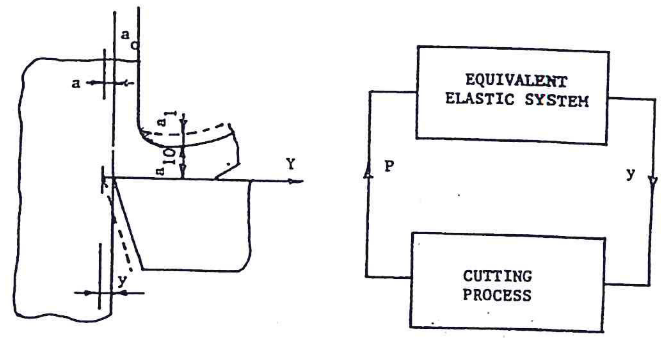
Relationship between the height of built up edge and the cutting speed (2) in turning steel 45

FIGURE 1



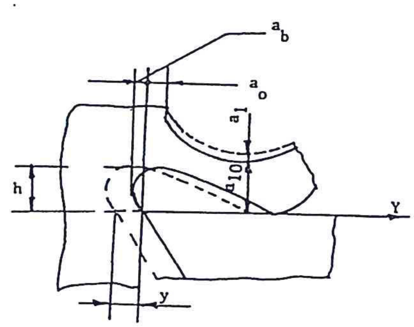
Relationship between built-up edge height and undeformed chip thickness in turning steel 45 (2)

FIGURE 2



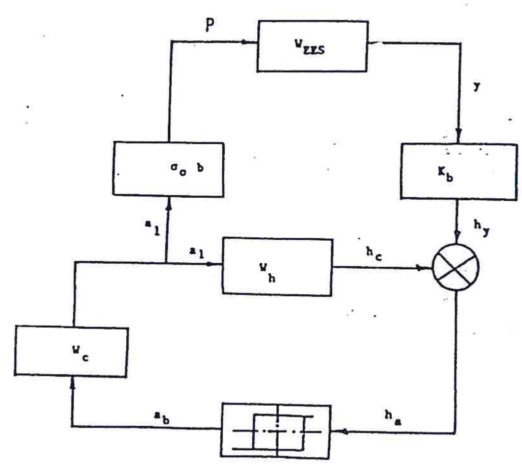
Dynamic system of metal cutting without built-up edge

FIGURE 3



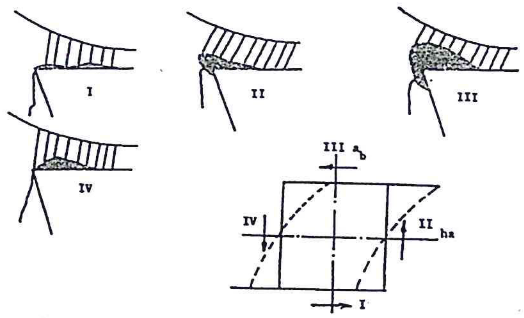
Schematic diagram of built-up edge formation in cutting steel

FIGURE 4



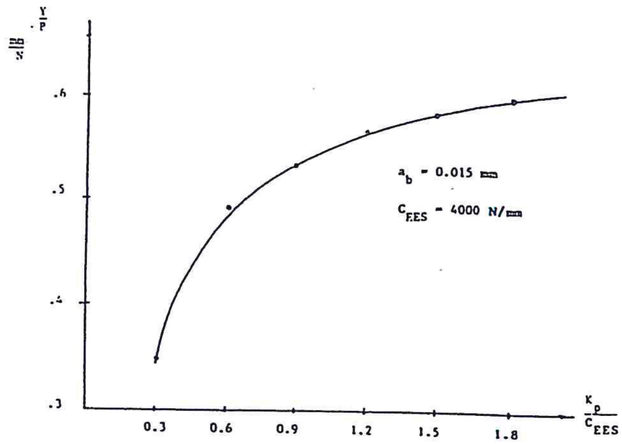
Dynamic system of a machine tool while cutting with a built-up edge

FIGURE 5



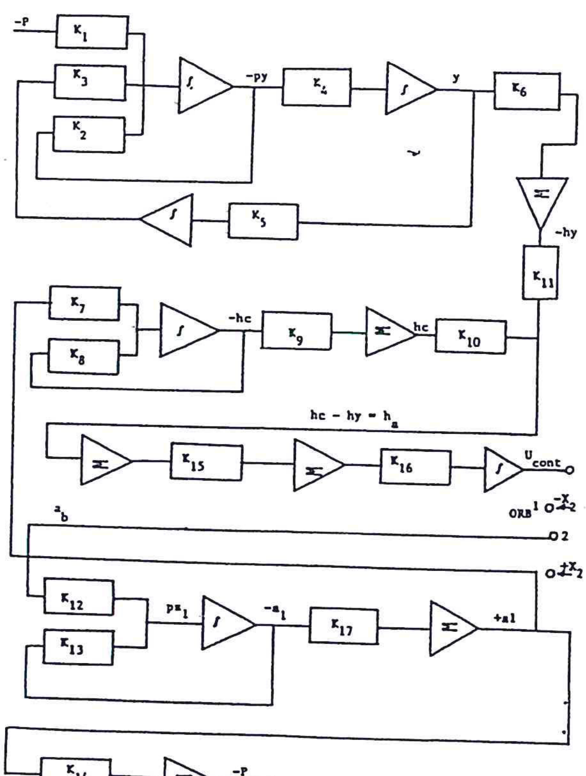
Relationship between the projecting part a_b of the built-up edge and its height h_a on the stages I to IV of the formation and rupture of the built-up edge

FIGURE 6



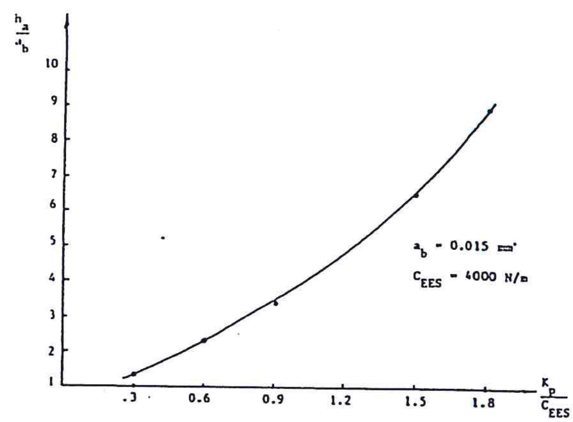
Relationship between $\frac{Y}{P}$ with $\frac{K_p}{C_{EES}}$

FIGURE 8(A)



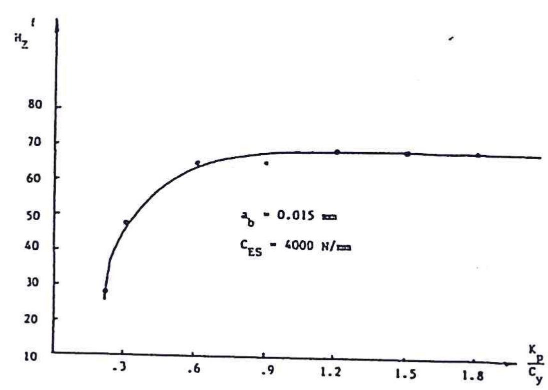
Block diagram of the nonlinear model

FIGURE 7



Relationship between $\frac{h_a}{a_H}$ and $\frac{K_p}{C_{EES}}$

FIGURE 8 (B)



Relationship between frequency of self excited vibration and $\frac{K_p}{C_y}$

FIGURE 8 (C)