COMPUTER PROGRAMS FOR SEISMIC INTERPRETATION IN FOUNDATION STUDIES

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SUMMARY

Computer programs for use, particularly in Civil Engineering studies, have been developed. They generate seismic time-distance curves for the single and multiple reflections, critical refractions, etc., that can arise in a medium of one or two uniform layers overlying a uniform substratum. The layer interfaces are specified numerically and may have any two-dimensional shape. Travel times and ray paths are computed according to the laws of geometrical optics, with an arbitrary headwave refraction hypothesis being used for the generation of headwaves. A number of program-produced ray path diagrams are displayed together with their corresponding time-distance plots.

1. INTRODUCTION

In Civil Engineering studies, the seismic method is frequently used to identify the lithologic units and their spatial configurations beneath the overburden. This is done by studying the time of travel of seismic disturbances through the ground. These stress perturbations are subject to reflection, refraction and "head-wave" refraction before returning to the surface. By noting the time of arrival of the 'echo' at different distances from the seismic source, a plot of travel-time versus distance can evince the spatial determination of the various geologic facies present under the area of investigation.

If it can be assumed that the propagation of the disturbances in the form of body waves conforms to the laws of geometrical ray optics, then for simple geologic models the theoretical time-distance curves can be computed analytically. For example, consider the boundary between two horizontally stratified layers in infinite halfspace - Figure 1. Let there be a reflection of the body wave at the boundary and let the wave propagate through the upper layer with velocity V. According to ray optics, the travel-time T of the ray from O to A is

$$T = (X^2 + 4H^2)^{\frac{1}{2}}/V$$

where X is the point of arrival of the ray measured horizontally from the origin of the pulse (disturbance) and H is the length to the boundary. This relation between T and X yields the theoretical time-distance curve.

In seismic travel-time interpretation, the field curves are compared with those corresponding to models for which the theoretical curves can be determined. If the correlation is high then the existing structure is assumed to be similar to that of the model. However, the models used are usually very simple in that they permit a simple algebraic calculation of the time-distance curve. What happens if the spatial configuration is complex? Interpretation becomes difficult as any analytical solution is intensive if not impossible.

This paper describes a method to produce time-distance curves as well as ray path diagrams for a small number of homogeneous isotropic layers overlying an infinite half-space where the interfaces between the layers are of arbitrary cylindrical shape.

Numerical rather than algebraic techniques are employed. In this way, preconceived ideas of possible structural attitudes can be tested in a relatively short time.

2. BODY WAVES AND RAY THEORY

There are two types of body waves which can exist in an elastic solid, namely, compressional (P) and shear (S) waves. These waves travel with velocities characterised by the elastic constants of the earth materials. The velocity V_p, of the *P*-wave is greater than the velocity V_s, of the shear wave.

Shear waves are not considered in this paper as they are not normally used in Civil Engineering studies. Apart from the fact that *P*-waves travel faster and therefore arrive before *S*-waves, there is a preferential generation of compressional waves near an explosion so that *P* waves dominate over the *S* waves. While it is possible for the *P* waves to be partially converted at a boundary of an elastic discontinuity into *S* waves, the conversion energy ratio is usually small. Also, most conventional detectors (geophones) are sensitive to vertical ground motion which permits some discrimination against the *S* waves.

The *P* wave on reaching a discontinuity in acoustic impedance (the product of the density and velocity of the earth material) becomes reflected and refracted according to Snell's law. This law states that any ray path obeys the relation

$$(\sin i)/V = a\ constant \qquad (1)$$

where i is the angle of incidence and V is the velocity of the wave-transmitting medium. For refraction at the interface between two different media, Snell's law becomes

$$(\sin i_1)/(\sin i_2) = V_1/V_2 \qquad (2)$$

where i_1 and i_2 are the angles between the normal to the interface and the rays in the media with velocities V_1 and V_2 respectively. Ray theory also says that for reflection at an interface,

$$\sin i = \sin r \qquad (3)$$

where r is the angle of reflection.

There is a critical angle, similar to that in geometrical optics, for which the refracted ray travels parallel to the interface. This implies that the ray must travel from one medium towards one having a higher velocity. If in equation (2), V_2 is greater than V_1 , and i_2 equals 90 degrees then the critical angle i_c is given by

$$Sin\ i_c = V_1/V_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

and the refracted wave travels with velocity V_2.

3. THE PROGRAMS

Three ray programs were developed dealing with:

(a) Reflection
(b) Refraction and reflection
(c) "Head-wave" refraction

A maximum of two boundaries, that is three layers, is considered. More boundaries may be considered using the same logical procedures but considerably more computer time is required. In all the cases studied the velocity of the layers increased with depth as is usually the case in nature. Program (a) deals with reflections from the uppermost boundary. Program (b) deals with reflection from the upper boundary when the critical angle of incidence is exceeded at the interface. It also deals with refraction from the first boundary followed by simple reflection from the second. Program (c) deals with head-waves generated by the first or second interface.

Programs (a) and (b) require only a straightforward application of Snell's Law. However, Snell's law cannot be used in Program (c) since geometrical optics does not predict the occurrence of head-waves. Rather it shows that if a ray (plane-wave) is incident at a boundary separating two layers at the critical angle i_c, there results only one ray which travels in a direction tangential to the boundary. If the boundary is flat it travels along the underside of the boundary. Plane-wave ray treatment indicates that no energy can be transmitted to the surface. However, it is known that waves do arrive at the surface carrying measurable energies when critical refraction takes place. The explanation of this paradoxical situation lies in the theory of curved wavefronts (1) where it is evident that a wavefront travelling on the underside of a boundary is capable of generating a wavefront of appreciable amplitude on the upper side. These wavefronts, called "head-waves" propagate upwards with ray paths inclined at the same angle i_c to the normal as are the down-going rays which are critically refracted. Figure 2 shows the ray paths of the head-waves for a flat horizontal boundary or interface.

Consider a boundary that is not flat but is of arbitrary shape as shown in Figure 3. PA represents the critically refracted ray. This ray does not travel exactly along the underside of the boundary except at P. It is possible for this ray to generate head-waves in the upper layer when it is travelling close to the boundary. How close the ray must be to the boundary to generate head-waves of appreciable amplitude can be determined using spherical wave theory. However, the complicated shape of the boundary makes the application of wave theory exceedingly difficult.

It is reasonable to assume that if the angle ψ (Figure 3) is small, the wavefront in the lower layer will produce Huygen-wavelets near the boundary which will generate perturbations in the form of head-waves in the upper layer. Program (c) uses an arbitrary criterion. Whenever angle ψ along the boundary is less than 5 degrees, "head-wave" refraction is assumed to take place. If the criterion is satisfied for the point Q say, then a head-wave leaves Q having a ray path inclined at an angle i_c to the normal at Q. The time interval for the wave to travel from P to Q is assumed to be (length of arc $PQ)/V_2$.

4. NUMERICAL DEFINITION OF AN ARBITRARY BOUNDARY

The assumed boundary is usually hand drawn by the interpreter. It can then be digitized and the data input to the computer. An interpolation routine must then be established to calculate the depth of the boundary at any horizontal position and also the gradient. It is imperative that the gradient be a smooth function of position. While many methods of smooth interpolation exist, few introduce no discontinuities in the derivatives. The most satisfactory method consists of using third order polynomials to interpolate each interval between data points such that the boundary passes through the end points of the interval and has specified gradients there. Gradient discontinuities can therefore not occur in passing from one interval to another. It is found that this interpolation method produces curves almost identical to those which are drawn by a person graphically interpolating the data.

Consider two adjacent data points x_i, $z(x_i)$ and x_j , $z(x_j)$ on a boundary. Let the gradients at x_i and x_j be $z'(x_i)$ and $z'(x_j)$ respectively. Choose a new x variable x^* such that

$$x^* = (x - x_i)/(x_j - x_i) = (x - x_i)/\Delta \quad \ldots\ldots\ldots\ldots (5)$$

where x is a value between x_i and x_j and

$$\Delta = x_j - x_i \quad \ldots\ldots\ldots\ldots (6)$$

Then $$\frac{dz}{dx^*} = z'\Delta \quad \ldots\ldots\ldots\ldots (7)$$

Let us find a third degree polynomial such that

$$z = a + bx^* + cx^{*2} + dx^{*3} \quad \ldots\ldots\ldots\ldots (8)$$

Using equations (7) and (8)

$$z'\Delta = b + 2c\,x^* + 3d\,x^{*2} \quad \ldots\ldots\ldots\ldots (9)$$

When $x^* = 0,\quad x = x_i$

$x^* = 1,\quad x = x_j$

Hence from equations (8) and (9)

$$a = z_i$$

$$b = z'_i \Delta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

$$a + b + c + d = z_j$$

$$b + 2c + 3d = z'_j \Delta$$

Solving equation (10), we obtain

$$a = a_{ij} = z_i$$

$$b = b_{ij} = z'_j \Delta$$

$$c = c_{ij} = 3(z_j - z_i) - (z'_j + 2z'_i)\Delta \quad \ldots\ldots\ldots \quad (11)$$

Knowing Δ, z_i, z_j, z'_i and z'_j from direct measurements on the plotted curve the coefficients a_{ij}, b_{ij}, c_{ij} and d_{ij} can be calculated.

It was found that eleven points or less were adequate to specify completely the boundaries studied. Δ was kept constant in all the programs i.e. the x - spacings between adjacent points were equal. This simplified the calculations.

Having once determined the coefficients of equation (11) for the specified points, equations (8) and (9) were used throughout the ray tracing programs to evaluate all boundary points together with their corresponding gradients.

5. PROGRAM ILLUSTRATIONS

The actual programs and flow charts will not be given since they will not contribute very much to the understanding of the principles underlying the programs. Rather the following examples will be used to illustrate the operational precepts of the three programs:

(a) Reflection from a single boundary - "REFLECTION" program.

Consider a ray making an angle *A1* with *OX* and striking the boundary at $P(x_1, z_1)$ - Figure 4. The reflected ray then emerges at the surface (assuming that it is not re-reflected for simplicity) making an angle β with *OX*. It can be shown that if the tangent at the point *P* makes an angle α with *OX* such that tan α is negative,

$$\beta = 2\alpha - A1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

If tan α is positive,

$$\beta = 2\alpha - A1 - 180^{\circ} \quad \ldots\ldots\ldots\ldots \quad (13)$$

The Slope *F* of the reflected ray is,

$$F = \tan\beta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

Let the equation of the boundary be,

$$z = z(x)$$

Then the slope of the tangent at *P* is,

$$z'(x) = \tan\alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (15)$$

Now *A1* can be calculated from the relation,

$$\tan A1 = z_1/x_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

Knowing *A1* and α, equations (12) and (14) can be used to determine *F*.

The equation of the reflected ray is then,

$$z = F(x - x_1) + z_1 \quad \text{.................} \quad (17)$$

Let $x = x_r$ where the ray arrives at the surface. Therefore,

$$x_r = x_1 - z_1/F \quad \text{.....................} \quad (18)$$

The respective lengths L_i and L_r of the incident and reflected rays are,

$$L_i = (x_1^2 + z_1^2)^{\frac{1}{2}} \quad \text{.....................} \quad (19)$$

and $L_r = \{(x_1 - x_r)^2 + z_1^2\}^{\frac{1}{2}}$ .... (20)

Thus L_i and L_r can be computed.

If V_1 is the velocity of the ray through the upper layer and T the time of travel, then

$$T = (L_i + L_r)/V_1 \quad \text{...................} \quad (21)$$

Equations (18) and (21) furnish the point of arrival of the ray and the time spent in the layer. Computing T and x_r for many values of x, a time-distance curve can be plotted. Logic to check that the rays L_i and L_r do not intersect the boundary at points other than (x_1, z_1) must of course be added. This was done so as to encompass all possible ray paths essential to the interpretation of the recorded field data.

(b) Refraction and reflection from two boundaries –

"REFRACTION AND REFLECTION" program.

Two boundaries are considered. The function of this program is to deal with refraction, reflection and "head-wave" refraction which may occur at the first boundary and only reflection at the second boundary.

For simplicity consider a ray which is refracted at P, reflected at Q, and refracted at P' before emerging at the surface - Figure 5. By determining the gradient at P and knowing $A1$, the angle of incidence i can be obtained. Snell's law then yields the angle of refraction r.

$$Sin\ i/Sin\ r = V1/V2 \quad \ldots\ldots\ldots\ldots \quad (22)$$

Knowing r, the equation of the ray PQ can be determined. The point of intersection Q can now be obtained by numerically solving the equation of the ray PQ with that of boundary 2. By a process similar to the one given in the "REFLECTION" program, the equation of the reflected ray QP' is obtained. This equation is then solved with that of boundary 1 to obtain the point P'. The angle of incidence i' is then computed to give r' from

$$Sin\ i'/Sin\ r' = V2/V1 \quad \ldots\ldots\ldots \quad (23)$$

Knowing r', the point of emergence O' on the surface can be determined.

Since the co-ordinates of O, P, Q, P' and O' are known together with the velocities $V1$ and $V2$, the total time of travel can be calculated.

When r equals 90 degrees, the critical angle i_c, is reached and "head-wave" refraction takes place at boundary 1. This effect will be considered in more detail in the next program.

When $i > i_c$, reflection takes place at boundary 1 and the calculations become as in (a) above.

(c) "Head-Wave" refraction from either of two boundaries -

"HEAD-WAVE REFRACTION" program.

Starting with the incident ray *OM* having an angle *A1* close to 90 degrees (Figure 6), the refracted ray *MN* from boundary 1 is determined. The ray *MN* is next refracted at the second boundary to produce the ray *NE* making an angle ϕ with the normal at *N*. If ϕ is less than 85 degrees (an arbitrary criterion) then ray *NE* is investigated to see whether it intersects or grazes the second boundary. If it intersects, no further calculations are made as no refractions from layer 3 to layer 2 are allowed to take place. All controls then return to the beginning of the program where another incident ray *OM* having an angle less than *A1* is considered. However if *NE* grazes, as at the point *D* in Figure 6, head-wave refraction is assumed to occur and the appropriate procedure is applied.

If *Q* is a point such that $85^{\circ} \leq \phi \geq 90^{\circ}$, the angle of incidence becomes approximately equal to i_c, where

$$Sin\ i_c = V2/V3 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (24)$$

Head-wave refraction is then assumed to originate from the point *Q*. The first head-wave refracted *(HWR)* ray *QP'* makes an angle $\theta = i_c$ with the normal. *QP'* strikes boundary 1 at *P'* where it is refracted to *O'*. *OO'* as well as the travel time can be calculated.

The other *HWR* rays are obtained by considering points adjacent to *Q*. For example, consider a point *Q'* adjacent to *Q*. For an *HWR* ray to leave this point an arbitrary decision is made. If the angle between *QQ'* and *QR* is say, less than 5 degrees, then head-wave refraction occurs. The length of the boundary curve *QQ'* can be calculated and the total travel time *T* derived from:

$$T = (OP + P''O'')/V1 + (PQ + Q'P'')/V2 + (arc\ QQ')/V3 \quad \ldots. \quad (25)$$

In this manner adjacent points may be investigated for head-wave refraction effects.

From Figure 6 it is clear that for angles of incidence just greater than '*i*', head-wave refraction cannot take place from the second boundary. Total reflection occurs. As '*i*' is increased or *A1* decreased, a critical value will be reached resulting in *HWR* effects from the first boundary. The points of arrival and travel times are then evaluated as discussed above.

The head-wave refraction program permits multiple reflection and multiple refraction of a single *HWR* ray. In this program, an *HWR* ray, *Q'S*, (Figure 7) is ignored if head-wave refracted again at *S*. This was done for one main reason: The *HWR* ray *S* has relatively small energy and for this energy to be distributed from *S* would mean that rays having quite small energies would arrive at the ground surface. These "small-energy" rays would be comparable to the noise level and therefore cannot be detected without considerable difficulty.

Consider the first boundary shown in Figure 7. Provided the criterion for ϕ is satisfied, head-wave refraction takes place and rays *A*, *B*, *C*, *D*, *E* and *F* are obtained. In the "REFRACTION" and "REFLECTION" program, rays such as OP_o, OP_1 and OP_2 are doubly refracted so that they emerge along paths D', E' and F'. If the points P_1 and P_2 are close to P_0, then the rays D, E and F do not deviate much from the corresponding *HWR* rays D, E and F. In this case only A, B and C should be termed the *HWR* rays since they arrive at O_1, O_2 and O_3 with much less energies than the rays which arrive at O_4, O_5 and O_6. A superposition of the time-distance graphs obtained from programs (b) and (c) can aid in the identification of the "truly" head-wave refracted rays.

6. PROGRAMMED EXAMPLES

The computation and plotting for all examples were done using the Computer and Plotter.

Initially the programs used boundaries described by sinusoidal functions. This was done mainly for "debugging" purposes. Figure 8 displays ray paths and time-distance plots for reflections and both refractions and reflections from sinusoidally varying boundaries.

The remaining diagrams were obtained using numerical definitions of the boundaries and layers having, in order of depth, "velocities" 2, 3 and 6 units/sec. The detectors of energy were assumed to lie in the range 0 - 6 units so that any rays outside this limit were not delineated.

The examples illustrated are not of any immediate seismic interest but rather demonstrate the capabilities of the programs in which multiple reflections and refractions are possible.

Figure 9 illustrates reflection, Figure 10 refraction and reflection and Figure 11 head-wave refraction.

The average computation time per example was less than two minutes. The least time (½ - 1 min.) was taken by the reflection program. In each example, the first incident rays were directed to strike the first boundary at points having equal increments Δ in x. Δ was 0.05 for the "REFLECTION" and "REFRACTION and REFLECTION" programs, and 0.005 for the "HEAD-WAVE REFRACTION" program.

7. CONCLUSION

The programs can be adapted to deal with more than two arbitrarily specified boundaries. The logic although slightly more complex, can be handled without too much difficulty.

The programs are simple and of considerable use in the interpretation of the seismic data obtained in foundation studies for buildings, bridges, airstrips, and roads. They can also be of immense value in the identification of the deeper geologic structures in the earth's crust.

REFERENCE

(1) JEFFREYS, H., On Compressional Waves in Two Superposed Layers, Proc. Camb. Phil. Soc., 1926, 22, 472-81.

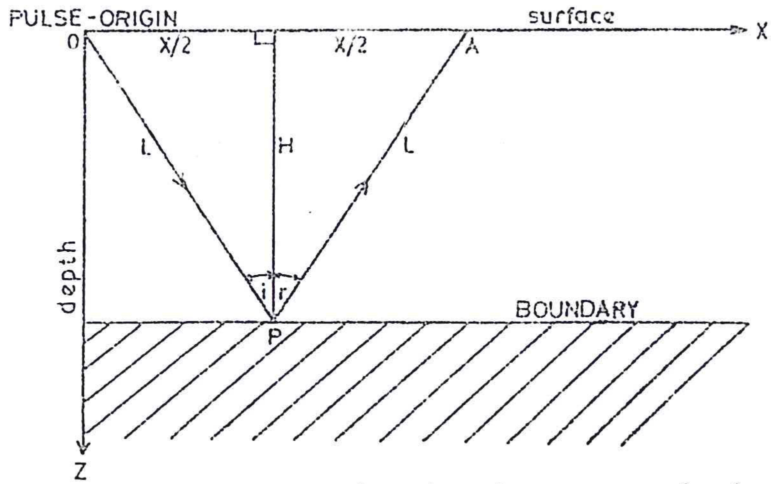


Fig. 1 - Reflection from the boundary between two horizontally stratified layers

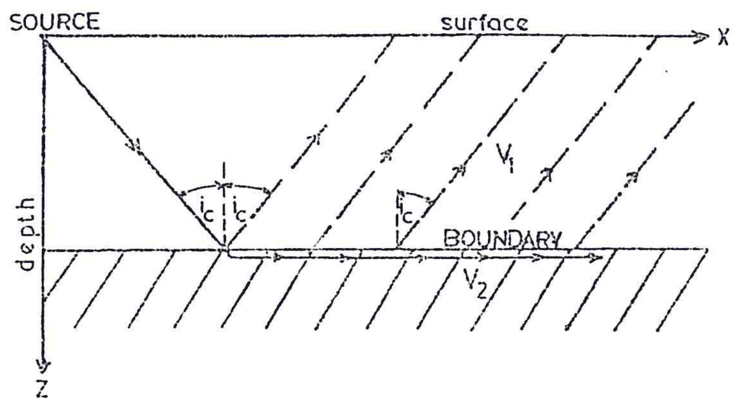


Fig. 2 - Critically refracted ray path (full line) and ray paths of the head waves (broken lines) for a flat boundary

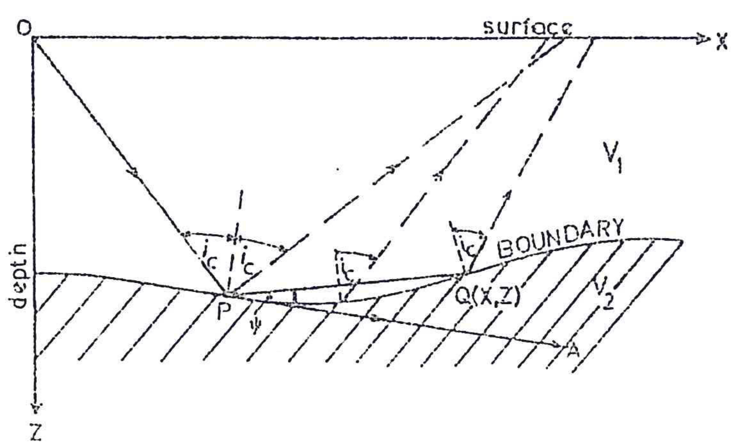


Fig. 3 - Critically refracted ray path and head wave ray paths, dependent on angle ψ

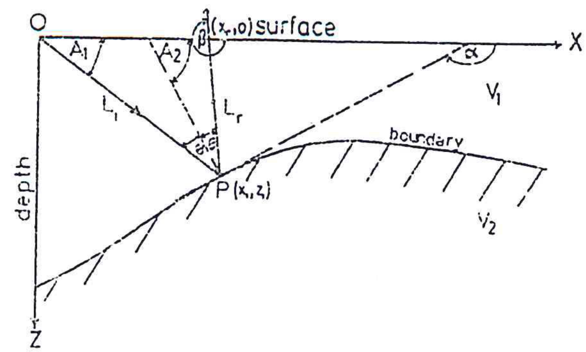


Fig. 4 - Reflection from a single boundary

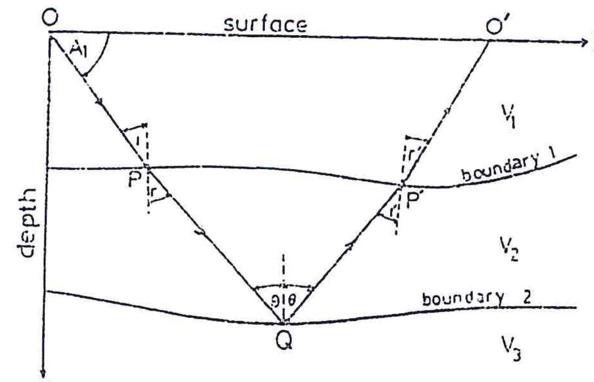


Fig. 5 -- Refraction and Reflection from two boundaries

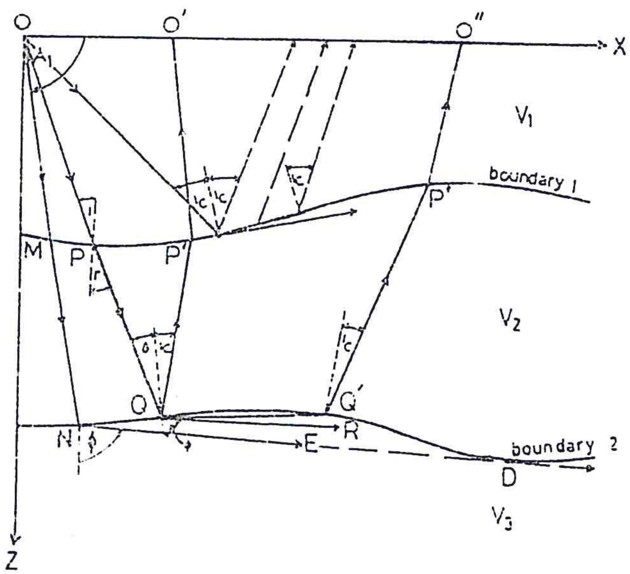


Fig. 6 - Head wave refraction from two boundaries

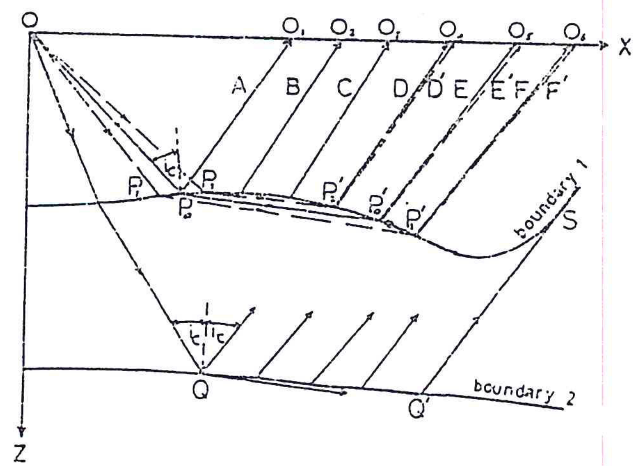


Fig. 7 - Head waves (full lines) and doubly refracted rays (broken lines)

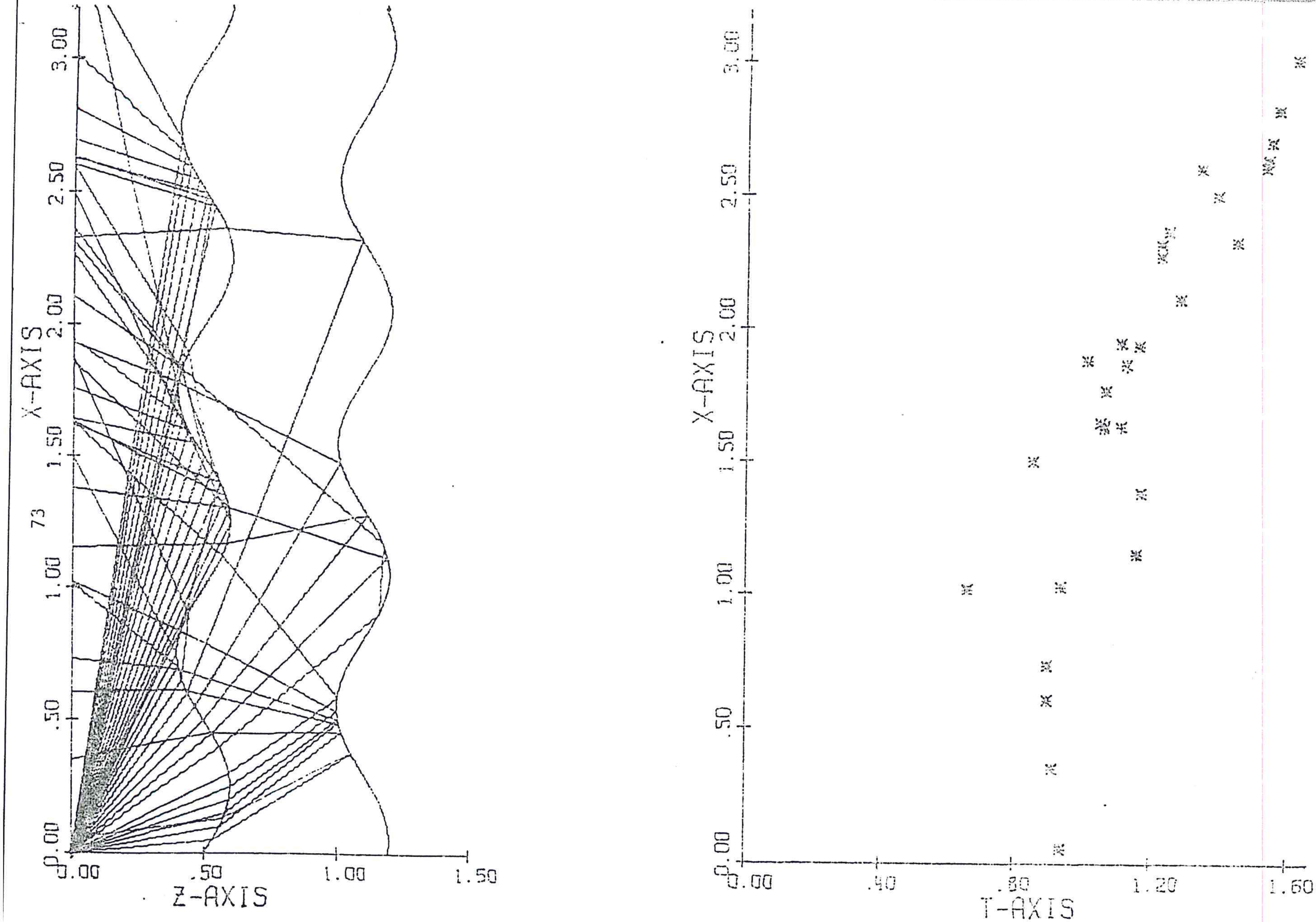


Fig. 8 - Ray Path Diagram and time-distance plot for refraction and reflection from sinusoidal boundaries

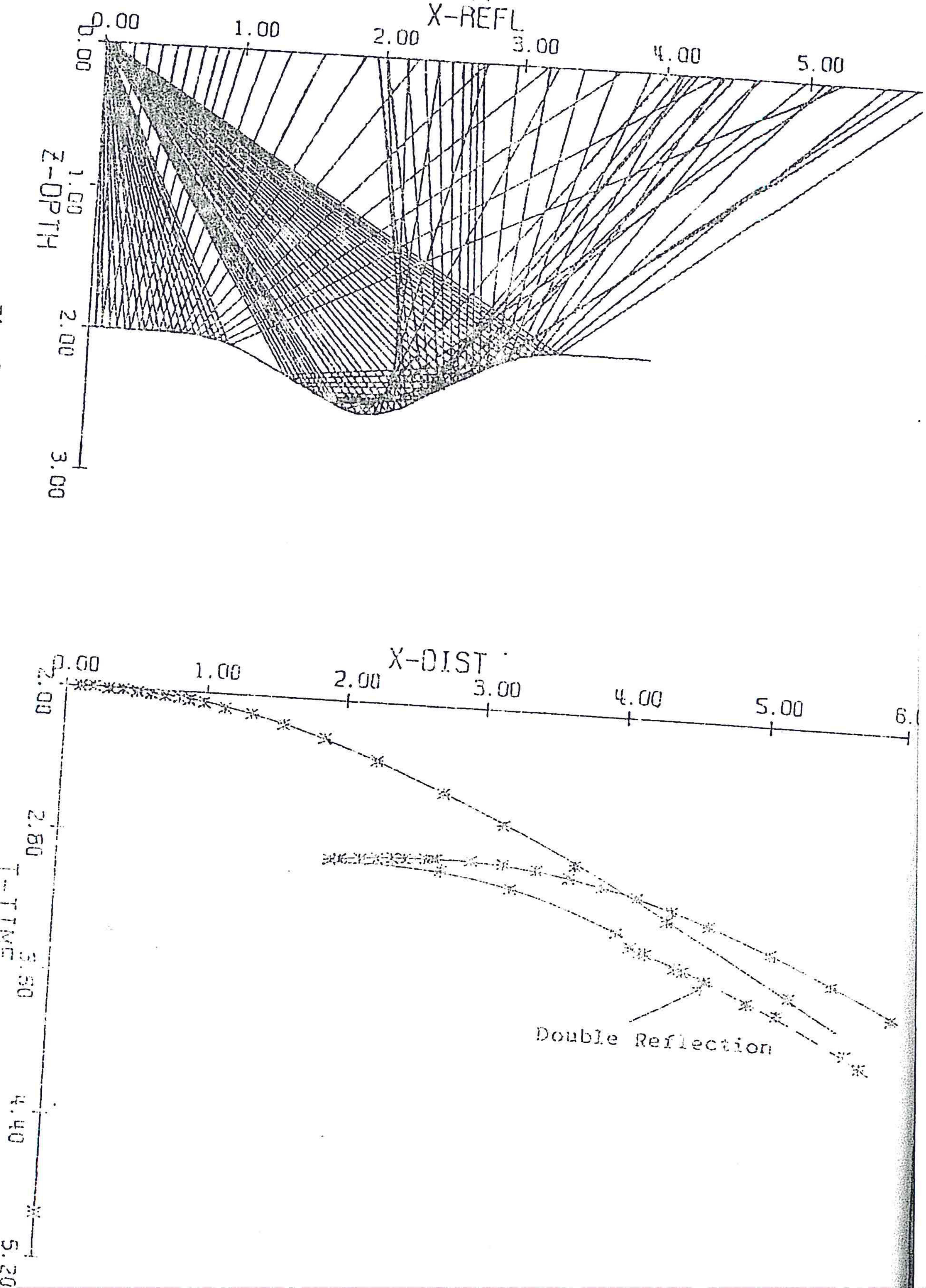
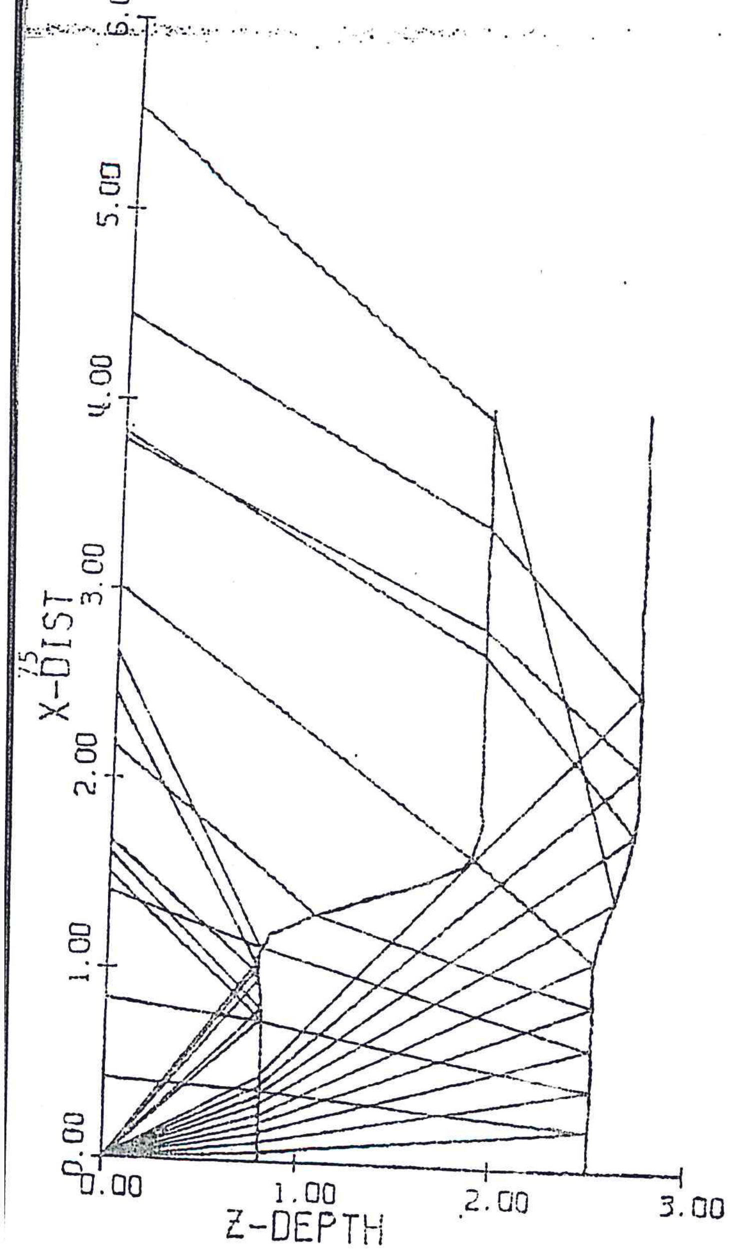


Fig. 9 - Reflection from a valley



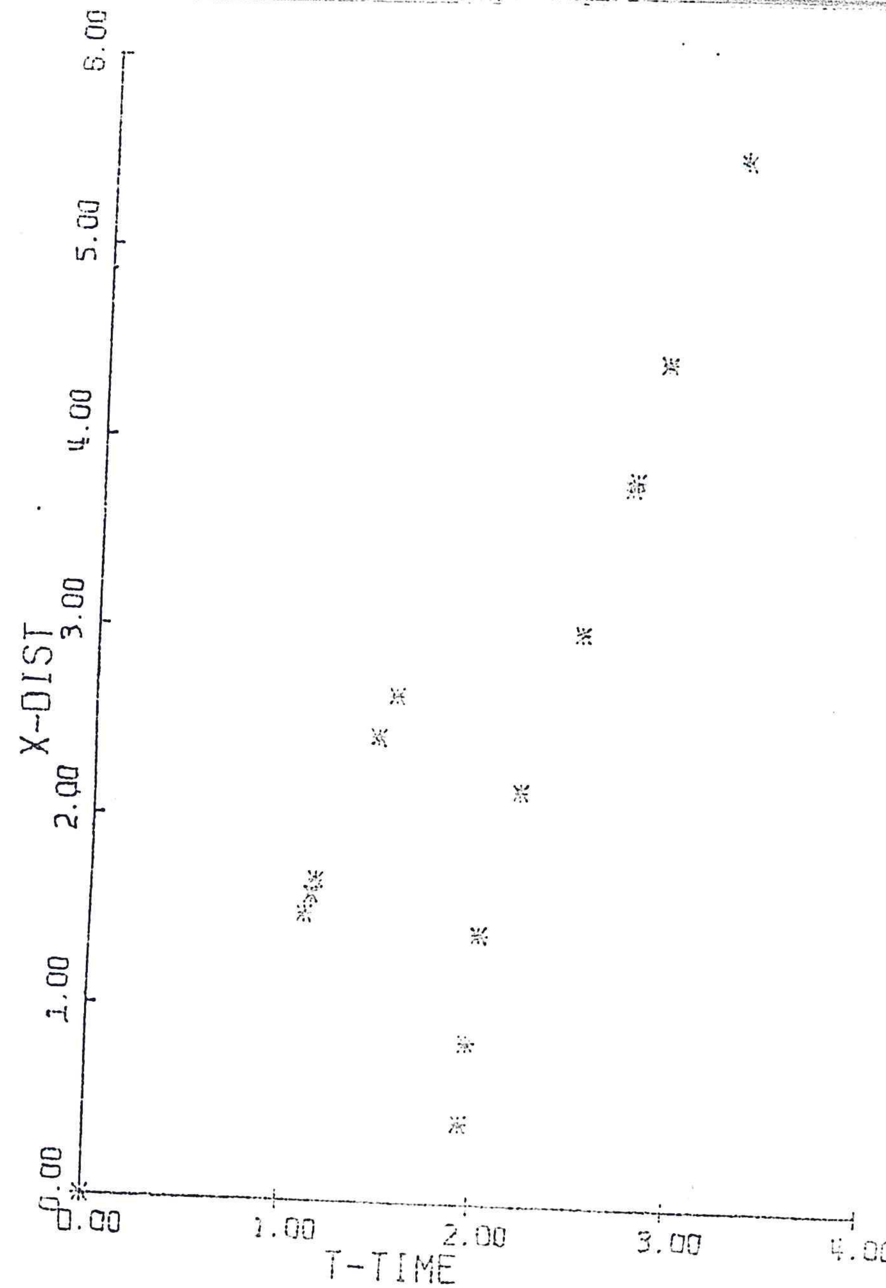


Fig. 10 - Refraction and reflection from a step boundary over a boundary with slight flexure

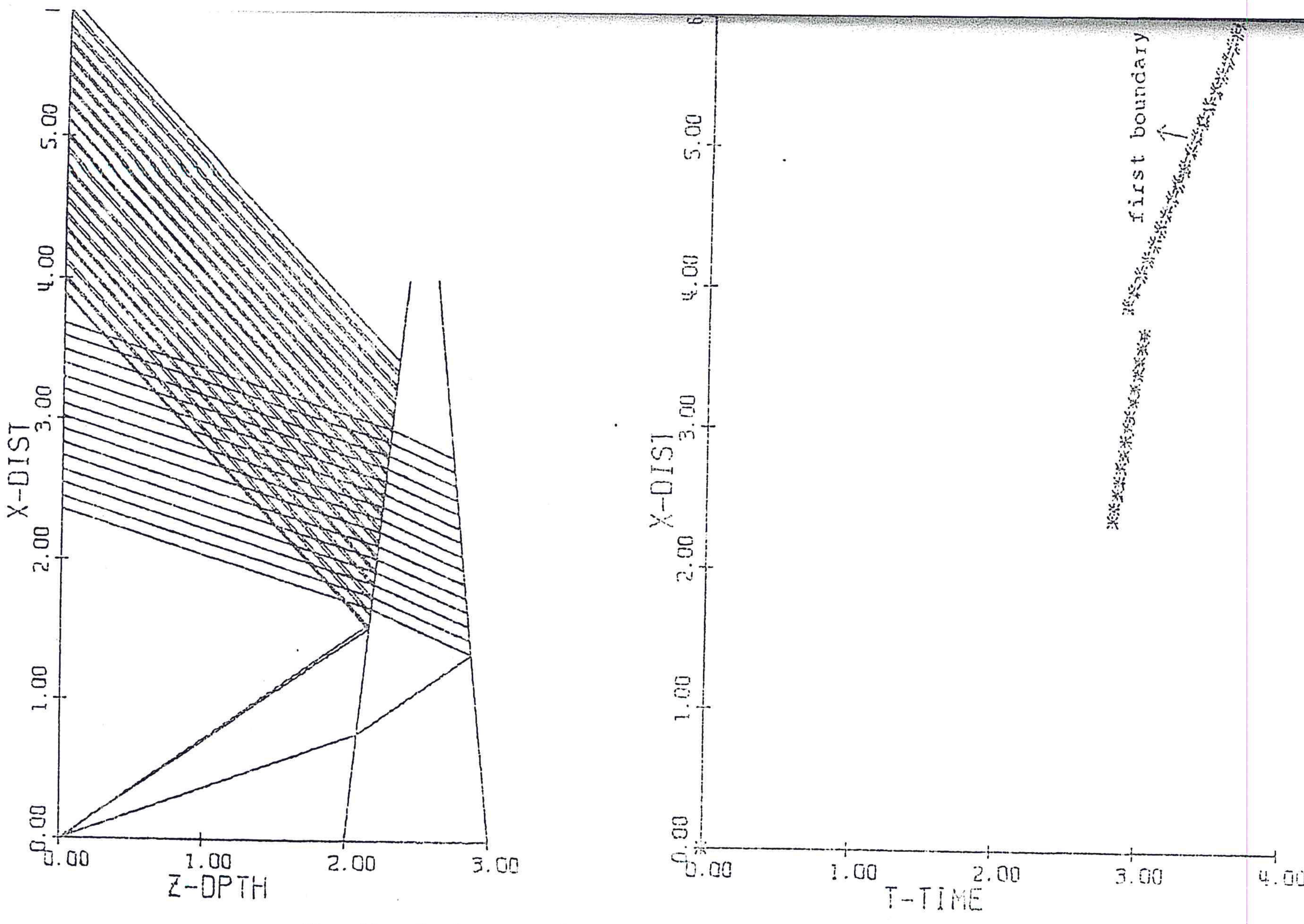


Fig. 11 - Head wave refraction from two tilting and plane boundaries. The HW refraction criterion is satisfied for two adjacent incident rays on the first boundary