A LINEAR SYSTEMS APPROACH TO TRANSPORTATION MODELLING

PART 1: TRAVEL DEMAND MODELLING IN A LINEAR SYSTEM

by

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SUMMARY

In the Capital Region of Trinidad and Tobago there exists no distinct pattern of zonal stratification from a transportation/land use viewpoint. It is necessary to develop a system of continuous travel demand modelling to replace discrete (zonal) modelling in this region. Such a modelling system was developed using gravity and opportunities models as basic tools. The linear system models proposed in this paper represent the first stage of this exercise. The proposed models were calibrated in Northern Trinidad, using 1973 and 1978 data, and conclusions were drawn about the models and model constants obtained. R^2 tests indicated significance levels of 99.9% for calibrated models (R^2 of 40% to 92%). Actual model constants were found to be related to both the mean trip length and the aggregation level observed in input data.

It was concluded that the models suggested the desirability of extending the linear system technique to more diverse land use patterns in an attempt to produce a practical and relatively cheap method of travel demand modelling.

1.0 INTRODUCTION

In recent times the worldwide growth of car ownership has led to extreme congestion in many of the world's cities. In an attempt to solve the many problems associated with this development the art of transportation planning has come into being. Essential to the transportation planning process is the rapidly developing technology of travel demand modelling.

Traditionally, the process of travel demand modelling consisted of four inter-related stages, with interactions among all stages being reflected in the iterative nature of the methodology (1). Data were traditionally stratified by a zonal system and the zones were treated as units throughout the process. Briefly, the stages may be described as follows.

1.1 Trip Generation

Trip generation is concerned with the decision of whether to make a trip or not. Accordingly, the trip generation model simulates the number of trips made from and to each zone.

1.2 Trip Distribution

At this stage trips leaving each zone are distributed to the other zones, based upon the properties of the zones, the trips and the tripmakers. The most common trip distribution models are the gravity models, the opportunities models and the utility models.

Gravity models are the most common distribution models. In its most used form (2) the model assumes the following equation:

$$T_{ij} = P_i \, A_j \, f(I_{ij}) / \sum_k A_k \, f(I_{ik}) \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

where T_{ij} = number of trips from zone i to zone j

P_i and A_j are number of trips produced in i and attracted by j respectively

I_{ij} = some measure of travel deterence or impedance between i and j, generally represented by time, cost, distance, etc.

A_k = attraction of zone k

k = some impedance function.

In many applications the impedance is expressed as travel time from zone i to zone j, t_{ij}. Equation 1 then becomes:

$$T_{ij} = P_i A_j f(t_{ij})/\Sigma A_j f(t_{ij}) \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

Model calibration is achieved by experimentally determining function f.

There are two types of opportunities models in use, the competing opportunities (3) and the intervening opportunities (4) models. The competing opportunities model assumes that:

$$T_{ij} = (P_i A_j/Ax_j)/\sum_k (A_k/Ax_k) \quad \ldots\ldots\ldots\ldots\ldots \quad (3)$$

where T_{ij}, P_i, A_j are as before and Ax_k is the number of opportunities up to and including the time band including zone j.

For this model, calibration is done by determining the separation of time bands drawn around the producer zone i.

In the intervening opportunities models, time bands are also drawn, and the model equation is:

$$T_{ij} = P_i \left\{\exp (LD_j) - \exp(L(D_j + A_j))\right\} \Big/ \sum_k \left\{\exp (LD_k) - \exp(L(D_k + A_k))\right\} \quad \ldots\ldots \quad (4)$$

where D_k is the number of intervening opportunities to zone k, corresponding to the number of available opportunities up to the time band before the one including zone k.

For this model the constant L is a probability constant to be determined from calibration on travel data.

Of the three model types mentioned, the utility models are the most recently developed. Based on economic utility theory, the most common form is the logit model (5) expressed by the equation:

$$T_{ij} = P_i \exp(U_{ij}) / \sum_k \exp(U_{ik}) \quad \ldots\ldots\ldots\ldots\ldots \quad (5)$$

where U_{ij} is the utility of choosing an origin-destination combination of i-j compared to all other i-k combinations.

The utility measure U_{ik} may be a combination of cost, time and other trip and tripmaker characteristics of the i-k journey, as well as the attraction of k, A_k. Calibration consists of determining this combination.

1.3 Modal Split

At the modal split stage trips are distributed among alternative modes of travel. This stage may be placed before or after trip distribution. Modal split models are commonly expressed by algebraic equations or modal diversion curves.

1.4 Route Assignment

Having simulated the number of trip interchanges from zone to zone by the modes available, these trips are then assigned to the road or transport network of the system. The procedure is based on a route assignment model and takes into account travel times and levels of congestion on the alternative routes from each zone to each other zone.

2.0 RESEARCH OBJECTIVES

In travel demand modelling in Northern Trinidad several major problems arose with the use of the traditional travel demand modelling methodology. Firstly, it was observed that the traditional zonal methodology required estimates of trip interchange data in the form of an origin-destination matrix. The collection of this data represents a costly component of the planning process and it was considered essential to develop a method of demand modelling capable of producing comparable accuracy levels at significantly reduced cost.

In many developing countries, the reduction of the data collection costs of the planning process represents a significant saving in the application of planning techniques to transportation services.

Two factors at work here are the financial burden of traditional planning techniques on poor countries, and the need for rapidly developing ones (such as Trinidad and Tobago) to make frequent travel analyses and predictions as travel/land use and general living conditions change.

It was furthermore observed that there are significant deficiencies in the zonal approach. In particular, travel variables, such as waiting times, travel times and costs, often vary significantly within any one zone, with the effect that intrazonal variables are often of the same order as interzonal variables. In Northern Trinidad there is an even more important deficiency in the zonal approach. By observation of the land use pattern within the Capital Region, it is obvious that, from a transportation point of view, there exists no distinct zonal stratification system. Land use is continuous in the region and it was therefore concluded that a continuous rather than a discrete (zonal) modelling approach was needed to represent the area.

Additional areas of concern in model calibration and analyses for Northern Trinidad were the use of alternative model types, model functions and travel periods. For 1973 data 24 hour travel was analysed. For 1978 data analyses were done for 06.00-10.00 hours and 06.00-18.00 hours travel. For all periods private and public transport were separately analysed.

3.0 THEORETICAL DEVELOPMENT

In developing the Linear Systems Technique two basic modelling approaches were initially considered (6). These are referred to as the sectional approach and the incremental approach and are described below.

3.1 Sectional Approach Model

Consider a land use system of negligible width and lateral travel compared to its length and longitudinal travel respectively. Such a system is referred to as a linear system, to which the urban area of Northern Trinidad approximates.

Figure 1 illustrates a linear system. Travel takes place across a Section S, and it is required to simulate the size of this flow at S, referred to as F_s. Tripmakers leave an element, dx, to make a trip to some destination. If x is less than s and y greater than s, then trips from dx to dy contribute to the flow F_s. Otherwise, they

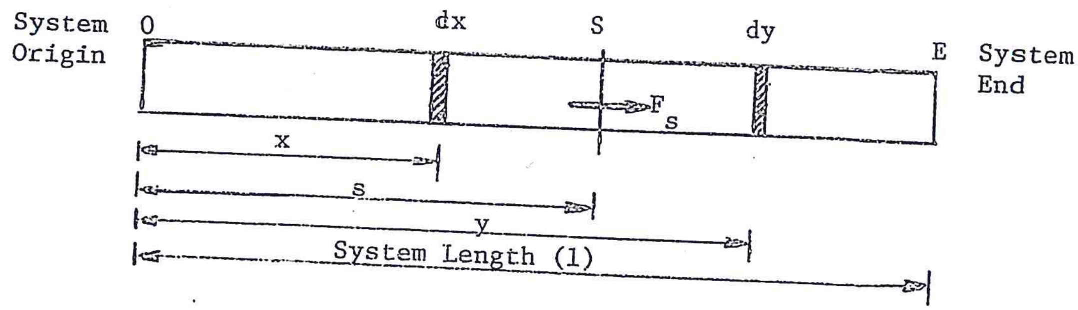

FIGURE 1 - SECTIONAL APPROACH MODELLING IN THE LINEAR SYSTEM

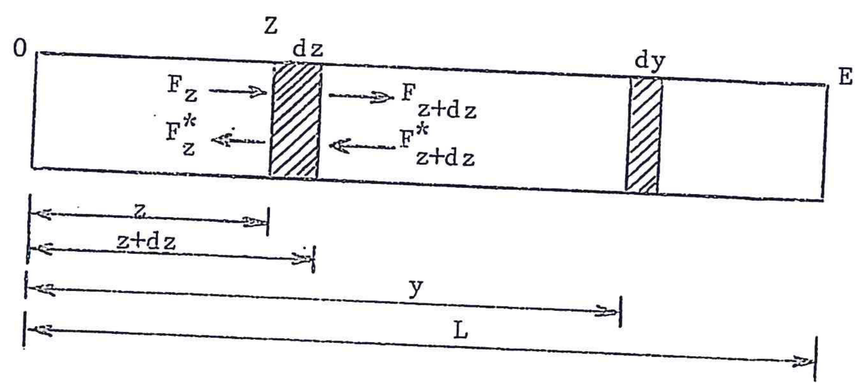

FIGURE 2 - INCREMENTAL APPROACH MODELLING IN THE LINEAR SYSTEM

do not contribute to that flow.

In estimating the number of trips made from dx to dy the gravity modelling form of Equation 1 may be used. The elements dx and dy may be treated as small 'zones', the size of which may be reduced indefinitely to reflect the most disaggregate level of the model. From Equation 1 the number of trips, $T_{dx \to dy}$, from dx to dy is given by:

$$T_{dx \to dy} = dPx\ f(I_{xy})\, dA_y / \sum_z f(I_{xz})\, dA_z \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

where dx produces dP_x trips and dy attracts dA_y trips.

I_{xy} = impedance from dx to dy and all other terms are as before.

For sufficiently small elements dx, dy, the flow F_s may therefore be evaluated by integrating with respect to dA_y between s and 1, and with respect to dP_x between 0 and s as in Equation 7.

$$F_s = \int_{x=0}^{s} \frac{\int_{y=s}^{L} f(I_{xy})\, dA_y}{\int_{y=0}^{L} f(I_{xy})\, dA_y} \cdot dP_x \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

Equation 7 may be used to generate alternative expressions for the flow F_s by varying the function f and impedance measure I_{xy}. For travel time impedance and the exponential impedance function exp(-wx), commonly used in gravity modelling, F, is given by Equation 8. Using this, the impedance from dx to dy is therefore given by $|y-x|$, used in Equation 8.

$$F_s = \int_{x=0}^{s} \frac{\int_{y=s}^{L} \exp(-w|y-x|)\, dA_y}{\int_{y=0}^{L} \exp(-w|y-x|)\, dA_y} \cdot dP_x \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

for a calibration constant w.

By introducing the substitution given by:

$$t = |y-x| \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (9)$$

and by noting that dA_y is zero for $y < 0$ or $y > L$, Equation 8 may be used to yield the alternative flow Equations 10 and 11.

$$F_s = \int_{x=0}^{s} \frac{\int_{t=s-x}^{\infty} \exp(-wt)\, dA_{x+t}}{\int_{t=0}^{\infty} \exp(-wt)\, d(A_{x+t} - A_{x-t})} \, dP_x \qquad \ldots\ldots\ldots \qquad (10)$$

This may be simplified to:

$$F_s = \frac{\int_{x=0}^{s} \int_{t=s-x}^{\infty} \exp(-wt)\, dA_{x+t}\, dP_x}{\int_{t=0}^{\infty} f(t)\, d(A_{x+t} - A_{x-t})} \qquad \ldots\ldots\ldots\ldots \qquad (11)$$

In general, for an impedance function, f, this modelling approach yields:

$$F_s = \frac{\int_{x=0}^{s} \int_{t=s-x}^{\infty} f(t)\, dA_{x+t}\, dP_x}{\int_{t=0}^{\infty} f(t)\, d(A_{x-t} - A_{x-t})} \qquad \ldots\ldots\ldots\ldots \qquad (12)$$

This model is referred to as the sectional approach time impedance model.

3.2 Incremental Approach Models

In Figure 2, a linear system is shown in which a flow F_z exists at section Z. Across the small element dz the flow changes to F_{z+dz}, by an amount dF_z. As tripmakers cross dz they are joined by a portion of the production of dz, dP_z. At the same time a portion of the attraction dA_z is 'filled' by tripmakers from the flow F_z. If the probability of a tripmaker at dz joining the flow in the direction of F_z is R_p, and the probability of a trip ending in dz coming from F_z is R_a, then the expected flow change dF_z is given by:

$$dF_z = R_p \, dP_z - R_a \, dA_z \qquad (13)$$

Before going on to consider the expressions for R_p and R_a it is useful to investigate the flow in the other direction, that is the reverse flow F^*_z. The flow F^*_z and F^*_{z+dz} are shown in Figure 2. Let the proportion of production dP_z joining F^*_{z+dz} to produce F^*_z be R^*_p, and the proportion of dA_z opportunities which are filled by flow from F^*_{z+dz} be the R^*_a. It follows that:-

$$-dF^*_z = R^*_p \, dP_z - R^*_a \, dA_z \qquad (14)$$

But:

$$R_p + R^*_p = R_a + R^*_a = 1 \qquad (15)$$

Combining Equations 13, 14 and 15 therefore yields:-

$$dF_z - dF^*_z = R_p \, dP_z + R^*_p \, dP_z - R_a \, dA_z - R^*_a \, dA_z = dP_z - dA_z \qquad (16)$$

At the extremities of the system the flow is zero. Integrating Equation 16 between 0 and z it follows that:-

$$F_z - F^*_z = P_z - A_z \qquad (17)$$

where P_z, A_z are cumulative productions and attractions from 0 to Z.

Equation 17 illustrates the fact that in a linear system the opposing flows are not mathematically independent. For sectional or incremental models then, it is necessary only to model the flow F_s (as in Section 3.1) and F^*_s may be obtained by the use of Equation 17.

3.2.1 *Gravity Model Type*

The factors R_p and R_a above are probability factors that may be determined by the use of an approach similar to that used for the sectional approach model. Consider the element dy in Figure 2. If dz and dy are treated as small 'zones' the probability Pr(z,y) of a trip from dz ending in dy is given by the gravity model as:-

$$Pr(z,y) = f(I_{z,y})dA_y / \int_{x=0}^{L} f(I_{z,x})dA_x \quad \ldots\ldots\ldots\ldots\ldots \quad (18)$$

The factor R_p is therefore given by integration of Equation 18 between Z and L:

$$R_p = \int_{y=z}^{L} f(I_{z,y})dA_y / \int_{y=0}^{L} f(I_{z,y})dA_y \quad \ldots\ldots\ldots\ldots\ldots \quad (19)$$

By a similar argument, the probability factor R_a is obtained as:

$$R_a = \int_{y=0}^{z} f(I_{y,z})dP_y / \int_{y=0}^{L} f(I_{y,z})dP_y \quad \ldots\ldots\ldots\ldots\ldots \quad (20)$$

As before, dP_y and dA_y are zero for $y < 0$ or $y > L$. Using line haul time for impedance and making the substitution:

$$t = |z-y| \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (21)$$

Equations 22 and 23 are derived:

$$R_p = \int_{t=0}^{\infty} f(t)dA_{z+t} / \int_{t=0}^{\infty} f(t)d(A_{z+t} - A_{z-t}) \quad \ldots\ldots\ldots \quad (22)$$

$$R_a = -\int_{t=0}^{\infty} f(t)dP_{z-t} / \int_{t=0}^{\infty} f(t)d(P_{z+t} - P_{z-t}) \quad \ldots\ldots \quad (23)$$

where t represents line haul travel time impedance from dz to dy.

Together with Equations 13 and 17, Equations 22 and 23 describe the flow in the linear system according to the travel time impedance model, incremental approach. The forward flow F_s is given by Equation 24.

$$F_s = P_s + \int_{z=0}^{s} \frac{\int_{t=0}^{\infty} f(t)\,dA_{z-t}}{\int_{t=0}^{\infty} f(t)\,d(A_{z+t} - A_{z-t})} \cdot dP_z \; +$$

$$\int_{z=0}^{s} \frac{\int_{t=0}^{\infty} f(t)\,dP_{z-t}}{\int_{t=0}^{\infty} f(t)\; d(P_{z+t} - P_{z-t})} \cdot dA_z \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (24)$$

3.2.2 *Intervening Opportunities Model Type*

The factors R_p and R_a discussed above may also be determined by the use of the intervening opportunities modelling approach. Figure 3 shows a linear system in which time bands are drawn around the element dz. Consider the elements at a time band t units from dz. According to the intervening opportunities model, the probability of a trip from dz ending in dz+t is Pr(z,z+t) where:

$$Pr(z,z+t) = \frac{1}{D}\left\{\exp(-w(A_{z+t} - A_{z-t})) - \exp(-w(A_{z+t} + dA_{z+t} - A_{z-t}))\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (25)$$

where A_{z+t}, A_{z-t} are cumulative attractions up to z+t, z-t as before, and D is a constant such that Pr(z,z+t) sums to unity over all elements dt of the system.

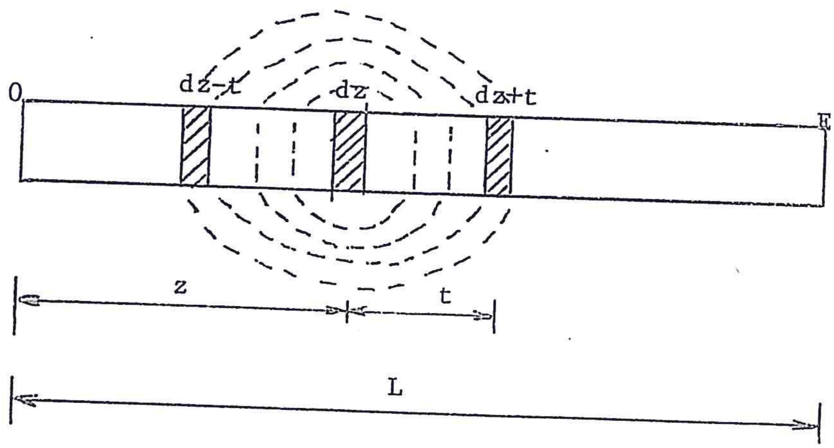

FIGURE 3 - TRAVEL TIME BANDS IN THE LINEAR SYSTEM

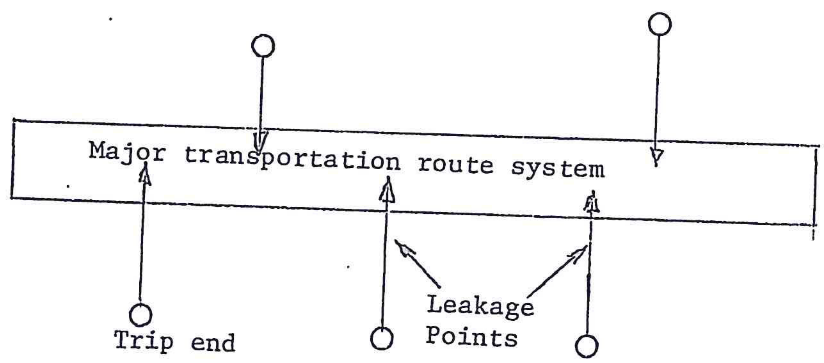

FIGURE 4 - TRANSLATION OF LEAKAGE POINTS TO LOCATIONS IN A LINEAR SYSTEM: INTIMATE SYSTEM LEAKAGE

Since $dz+t$ is a small element, it follows that dA_{z+t} is small compared to the total attraction of the system. The linear approximation for the experimental function may therefore be used in Equation 25 to yield:-

$$Pr(z,z+t) = \frac{1}{D} \exp(-w(A_{z+t} - A_{z-t}))(1 - \exp(-w\ dA_{z+t}))$$

$$= \frac{1}{d} \exp(-w(A_{z+t} - A_{z-t}))\ W\ dA_{z+t} \quad \ldots\ldots\ldots\ldots \quad (26)$$

For every element $dz+t$, a corresponding element $dz-t$ exists (which may be outside the system, in which case dA_{z-t} is zero). The value of D may therefore be determined as:

$$D = \int_{t=0}^{\infty} \exp(-w(A_{z+t} - A_{z-t}))wd(A_{z+t} - A_{z-t}) \quad \ldots\ldots\ldots\ldots \quad (27)$$

This leads to:-

$$D = \int_{u=0}^{A} \exp(-u)\ du = 1 - \exp(-wA) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (28)$$

where $u = w(A_{z+t} - A_{z-t})$ .... (29)

and A is the total attraction of the system.

Summing between dz and E, R_p is obtained as:

$$R_p = w \int_{t=0}^{\infty} \exp(-w(A_{z+t} - A_{z-t}))dA_{z+t}/(1 - \exp(-wA)) \quad \ldots \quad (30)$$

An alternative expression for R_p is obtained by noting that:

$$\int_{t=0}^{\infty} \exp(-w(A_{z+t} - A_{z-t}))d(A_{z+t} - A_{z-t})$$

$$= (1 - \exp(-wA))/w \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (31)$$

therefore,

$$R_p = \frac{\int_{t=0}^{\infty} \exp(-w(A_{z+t} - A_{z-t}))\,dA_{z+t}}{\int_{t=0}^{\infty} \exp(-w(A_{z+t} - A_{z-t}))\,d(A_{z+t} - A_{z-t})} \quad \ldots\ldots\ldots\ldots \quad (32)$$

similarly,

$$R_a = \frac{-\int_{t=0}^{\infty} \exp(-w(P_{z+t} - P_{z-t}))\,dP_{z-t}}{\int_{t=0}^{\infty} \exp(-w(P_{z+t} - P_{z-t}))\,d(P_{z+t} - P_{z-t})} \quad \ldots\ldots\ldots\ldots \quad (33)$$

Equations 32 and 33 exhibit a distinct similarity of form to Equations 22 and 23 for the time impedance model. The intervening opportunities model may therefore be seen as belonging to a class of 'gravity models' in which travel impedance is measured by the intervening opportunities ($A_{z+t} - A_{z-t}$ in this case) and the impedance function is negative exponential. This suggests the extension of the model to the use of a general function f, in which case R_p and R_a are given as:

$$R_p = \frac{\int_{t=0}^{\infty} f(A_{z+t} - A_{z-t})\,dA_{z+t}}{\int_{t=0}^{\infty} f(A_{z+t} - A_{z-t})\,d(A_{z+t} - A_{z-t})} \quad \ldots\ldots\ldots\ldots \quad (34)$$

$$R_a = \frac{-\int_{t=0}^{\infty} f(P_{z+t} - P_{z-t})\,dP_{z-t}}{\int_{t=0}^{\infty} f(P_{z+t} - P_{z-t})\,d(P_{z+t} - P_{z-t})} \quad \ldots\ldots\ldots\ldots \quad (35)$$

Equations 34 and 35, together with the basic equations 13 and 17, describe the opportunities impedance model. As an aid in simulation it is useful to note that the denominator in Equations 34 and

35 may be evaluated, for example:-

$$\int_{t=0}^{\infty} f(A_{z+t} - A_{z-t}) = \int_{u=0}^{A} f_1(u)du = f_1(A) - f_1(0) \quad \ldots\ldots \quad (36.)$$

where A = total attraction of the system and f_1 is the integral function of f.

3.2.3 *Generation Impedance Modelling*

The intervening opportunities to a destination are considered to be all the opportunities which are more easily accessible than the destination, from the trip origin (7). In Section 3.2.2, impedance is measured by the number of intervening opportunities. A further extension of the intervening opportunities model is made by expressing impedance as the opportunities actually passed on the way to the destination. These opportunities are 'physically' perceived as intervening, and may be the most significant intervening opportunities for non-routine trips, where the tripmaker does not have much detailed information of the system in relation to that trip type. This idea leads to Equations 37 and 38, obtained by replacing total intervening opportunities by those along the path of the trip.

$$R_p = \int_{t=0}^{\infty} f(A_{z+t} - A_{z-t})aA_{z+t} \Big/ \Big(\int_{t=0}^{\infty} f(A_{z+t} - A_{z-t})dA_{z+t}$$

$$- \int_{t=0}^{\infty} f(A_z - A_{z-t})dA_{z-t}\Big) \quad \ldots\ldots \quad (37)$$

$$R_a = - \int_{t=0}^{\infty} f(P_z - P_{z-t})dP_{z-t} \Big/ \Big(\int_{t=0}^{\infty} f(P_{z+t} - P_z)dP_{z+t}$$

$$- \int_{t=0}^{\infty} f(P_z - P_{z-t})dP_{z-t}\Big) \quad \ldots\ldots \quad (38)$$

Equations 37 and 38 are descriptive of the generation impedance model, derived as a special extension of the opportunities impedance model. The expressions may be simplified by evaluating the integrals as above. In combination with Equation 13, the flow F_S is obtained as:-

$$F_S = P_S - \int_{P_{z=0}}^{P_S} \frac{f_1(A_z) - f_1(0)}{f_1(A_z) + f_1(A-A_z) - z\, f_1(0)} \cdot dP_z$$

$$- \int_{A_{z=0}}^{A_S} \frac{f_1(P_z) - f_1(0)}{f_1(P_z) + f_1(P-P_z) - 2f_1(0)} \cdot dA_z \qquad \ldots\ldots\ldots\ldots\ldots \quad (39)$$

where P_S, A_S are cumulative production and attraction up to Section S.

P,A = total production and attraction of the system.

f_1 = the integral function of f, the impedance function.

Together with Equation 17, Equation 39, describes the generation impedance model.

3.3 Open Linear Systems

In the linear system described above, all trips had their ends within the system. This type of system is referred to as "closed". In an "open" system, trips have either two, one or no ends in the system. An open system is said to be experiencing 'flow leakage'. Such leakages are classified as 'intimate', 'isolated' or 'complex'.

3.3.1 *Intimate Leakage*

In intimate leakage the external trip ends are so close to the linear system that little error arises from treating these trip ends as points of trip generation at locations along the system.

Figure 4 illustrates the transformation which converts such an open linear system into a closed one.

3.3.2 *Isolated Leakage*

If external-internal trips are of sufficient length, it may be assumed that these trips are distributed to elements of the system in direct proportion to the generation of the elements and independently of their positions. In such a situation these external-internal trips are said to be generated at an "isolated generator", and isolated leakage is said to be taking place.

Flow F_S at S is broken down into two parts. $F_{i,s}$, the localised flow arises from external-internal trips while F_{lks}, the leakage flow, is externally generated. Let the cumulative leakages up to Z be P_{lkz} into the system and A_{lkz} out of it. Total leakages into and out of the system are P_{lk} and A_{lk} respectively. An element dz in the system (Figure 3) yields an amount of production $dP_z^{(lk)}$ and attraction $dA_z^{(lk)}$ for leakage trips, as given by:-

$$dP_z^{(lk)} = dP_z\, A_{lk}/P \qquad (40)$$

$$dA_z^{(lk)} = dA_z\, P_{lk}/A \qquad (41)$$

(all terms are previously defined)

The yielded amount generates leakage flow while the remainder produces localised flow $F_{i,s}$. Hence:-

$$F_{i,s} = \int_{z=0}^{s} dF_{i,z} \qquad (42)$$

where

$$dF_{i,z} = R_p dP_z(1-A_{lk}/P) - R_a dA_z(1-P_{lk}/A) \qquad (43)$$

The reverse localised flow $F_{i,s}^*$ is related to $F_{i,s}$ by Equation 44 (from Equation 17).

$$F_{i,s} - F_{i,s}^* = P_s(1-A_{lk}/P) - A_s(1-P_{lk}/A) \qquad (44)$$

Since a portion of the opportunities at each destination is filled by external trips, R_p and R_a are obtained from Equations 19 and 20 by replacing P_z by $(1-A_{1k}/P)P_z$ and so on to give:-

$$R_p = \int_{y=z}^{L} f(I_{z,y})d(A_y(1-P_{1k}/A))/\int_{y=0}^{L} f(I_{z,y})d(A_y(1-P_{1k}/A)) \quad \ldots \quad (45)$$

and similarly for R_a.

This equation reduces to Equation 19, illustrating the fact that R_p and R_a are unaffected by isolated leakage.

In crossing dz, leakage flow is changed by the yielded portion of generation at dz, as well as the actual leakage at leakage points between 0 and Z (Figure 3). The first mentioned part is $dF_{1kz}^{(i)}$, is given by:-

$$dF_{1kz}^{(i)} = R_p^{(1k)} dP_z^{(1k)} - R_a^{(1k)} . dA_z^{(1k)} \quad \ldots\ldots\ldots\ldots\ldots \quad (46)$$

where $R_p^{(1k)}$ and $R_a^{(1k)}$ are obtained from Equations 19 and 20 by replacing P_y by P_{1ky} and so on, and noting that I_{zy} is constant for isolated leakage.

Hence:-

$$dF_{1kz}^{(i)} = (1-A_{1kz}/A_{1k})dP_z(A_{1k}/P)$$

$$- (P_{1kz}/P_{1k})dA_z(P_{1k}/A) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (47)$$

(since $P_z^{(1k)}$ and $A_z^{(1k)}$ are obtained from Equations 40 and 41).

The second part, $dF_{1kz}^{(1k)}$ is obtained by assuming that external-internal or through trips are either insignificant or are analysed separately from the isolated leakage problem (these, in fact, do not literally represent any 'leakage'). Therefore,

$$dF_{1kz}^{(1k)} = R_p^{(i)} dP_{1kz} - R_a^{(i)} dA_{1kz} \quad \ldots\ldots\ldots\ldots\ldots \quad (48)$$

where $R_p^{(i)}$, $R_a^{(i)}$ are factors as before, for external-internal trips entering or leaving dz being distributed to the right or left of dz respectively (Figure 3). Hence,

$$dF_{1kz}^{(1k)} = (1-A_z/A)dP_{1kz} - (P_z/P)dA_{1kz} \quad \ldots\ldots\ldots\ldots\ldots \quad (49)$$

Continuing $F_{i,s}$, $F_{1ks}^{(i)}$, and $F_{1ks}^{(1k)}$, the flow F_s is obtained from:

$$dF_z = (1-A_{1k}/P)R_p dP_z - (1-P_{1k}/A)R_a dA_z$$

$$+ (1-A_{1kz}/A_{1k})dP_z (A_{1k}/P) - (P_{1kz}/P_{1k})dA_z(P_{1k}/A)$$

$$+ (1-A_z/A)dP_{1kz} - (P_z/P)dA_{1kz} \quad \ldots\ldots\ldots\ldots\ldots \quad (50)$$

This expression may be integrated between 0 and S to obtain F_s as:-

$$F_s = (1-A_{1k}/P) \int_{z=0}^{s} R_p dP_z - (1-P_{1k}/A) \int_{z=0}^{s} R_a dA_z$$

$$+ \frac{P_s}{P}(A_{1k} - A_{1ks}) + P_{1ks}\frac{(1 - A_s)}{A} \quad \ldots\ldots\ldots\ldots\ldots \quad (51)$$

By interpreting P_s and A_s in Equation 17 as total cumulative trips produced and attracted respectively, up to Section S, including internal and external trip ends, the reverse flow F_s^* is given by Equation 52.

$$F_s - F_s^* = (P_s + P_{1ks}) - (A_s + A_{1ks}) \quad \ldots\ldots\ldots\ldots\ldots \quad (52)$$

Equations 51 and 52 describe the open linear system, for isolated leakage.

3.2.3 *Complex Leakage*

In this category, leakage takes place between two (or more) systems of comparable size or variable separation, so that neither intimate nor isolated leakage theory approximates to the situation. In such cases the systems may be combined to produce a more complex system for which more advanced analysis is required. The relevant extensions of the Linear Systems Theory are to be presented in a sequel (8) to this paper.

3.4 Model Constraints

In developing the expressions for R_p and R_a, above, the basic gravity and opportunities models were used. Both models are singly constrained, satisfying the production constaint of Equation 53, but not in general satisfying Equation 54 (in contrast to Wilson's entropy model for example (9)).

$$\sum_j T_{ij} = P_i \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (53)$$

$$\sum_i T_{ij} = A_j \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (54)$$

for zones i, j, where T_{ij} = trips from i to j.

In gravity modelling a procedure is described for approximately satisfying both constraints simultaneously (see Hutchinson (1)). In the linear system the constraints to be satisfied are expressed by Equation 55 (See Figures 2,3).

$$F_0 = F_0^* = F_1 = F_1^* = 0 \qquad \ldots\ldots\ldots\ldots\ldots \qquad (55)$$

Since flow differences are measured from 0, F_0, and F_0^* are zero. However, in order to obtain zero flows F_1, F_1^* a correction procedure must be used (as is done in gravity modelling for example). If the simulated flow at system end E is $F_1^{(mod)}$, representing the end error in Equation 55, the corrected flow $F_s^{(cor)}$ is given in terms of the simulated flow $F_s^{(mod)}$ by:

$$F_s^{(cor)} = F_s^{(mod)} - F_1^{(mod)} \cdot G_s/G \quad \ldots\ldots \qquad (56)$$

where G_s is the cumulative generation up to S and G represents total generation.

Flow F_s^* may similarly be corrected.

4.0 MODEL CALIBRATION

The models presented in this paper were calibrated on data collected for the years 1973 and 1978 for the Trinidad and Tobago Capital Region. This area, from Arima to Diego Martin, approximates to an open linear system, and was analysed as such. Calibrations were performed by minimising the relative deviation statistic. Considering both private and public transport models the calibrations suggested three direct conclusions:

(i) The calibration constant varies with the level of disaggregation, increasing as the integral step width in the flow equations is decreased. A step width of 1/8 of the flow interval, corresponding to about 30 seconds travel time, was chosen for calibration, since this represented a likely level of valuation of travel time by tripmakers.

(ii) The calibration constant varies according to an inverse relation with the mean trip length, regardless of mode or time of day. This strongly suggests that models calibrated on travel data of a certain combination of trip types (and hence a particular mean trip length) should not be applied to a different trip type combination. This suggests that it is preferable to develop models on data disaggregated by trip purpose, and that model stability may not otherwise be possible.

(iii) Calibration constants were comparable to those obtained in zonal modelling in North American and British cities (1), (10), (11), (12). These constants are listed for the negative exponential function in Table 1, together with the corresponding R^2 values for the calibrations. Information given is for 1978 travel, 06.00-10.00 hours.

5.0 CONCLUSIONS

From the theoretical work described here it is clear that the linear systems technique is capable of generating considerable research material. In particular, it is necessary to extend the theory to deal with more complex systems. Such an extension is to be described in a sequel to this paper. The relatively high values of R^2 obtained from tests in Trinidad indicate that the techniques are potentially useful in the field of travel demand simulation. Since the models may be calibrated from traffic count data (F_s observed) this suggests a further advantage over the traditional modelling methodologies. In the meantime, research continues on the development and testing of a comprehensive linear systems technique for travel demand simulation in general land use systems in Trinidad and Tobago and other Caribbean countries.

	PRIVATE TRANSPORT (CAR)		PUBLIC TRANSIT (BUS & TAXI)	
	Calib. Const. (w)	R^2 Value	Calib. Const. (w)	R^2 Value
Sectional Approach, Time Impedance	0.07	0.92	0.07	0.85
Incremental, Time Impedance	0.12	0.66	0.13	0.42
Incremental, Opportunities Impedance	5.7×10^{-5}	0.82	6.5×10^{-5}	0.70
Incremental, Generation Impedance	6.1×10^{-5}	0.62	6.1×10^{-5}	0.45

TABLE 1 - CALIBRATION RESULTS FOR NORTHERN TRINIDAD LINEAR SYSTEM 0.600-10.00 HOURS TRAVEL USING NEGATIVE EXPONENTIAL FUNCTION

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